Is the Taylor Rule Nonlinear? Empirical Evidence from a Semi-Parametric Modeling Approach*

Nikolay Markov† and Carlos de Porres‡
University of Geneva
Department of Economics

First version: May 2011
Revised: January 2012

Abstract

This paper investigates the empirical specification of the Taylor Rule. Building on the New Keynesian paradigm, we derive a general nonlinear specification for the optimal policy rule that nests the standard linear Taylor Rule as a particular case. Using a novel semi-parametric modeling approach, we estimate both a standard and an augmented specification of the Taylor Rule for 8 OECD countries over the period 1976-2010 within a quarterly frequency. The augmented specification includes an interaction term between economic fundamentals and accounts explicitly for the interaction between economic fundamentals and the business cycle. The empirical results first point to a strong evidence for nonlinearity in the Taylor Rule for all countries. Second, our augmented specification outperforms both the standard linear and nonlinear Taylor Rules as it fits better the actual policy rate. Third, the performance of the augmented semi-parametric Taylor Rule is similar to the one of a linear policy rule with interest rate smoothing. An out-of-sample forecasting of the policy rate indicates that the nonlinear specification clearly outperforms the linear Taylor Rule for all countries using two different forecasting procedures and two measures of the economic outlook. The nonlinear interest rate rule thus provides an accurate guideline for the Central Bank decisions on the key interest rate, while at the same time avoids a possible misspecification problem that might arise when using interest rate smoothing in the policy rule.

JEL Classification: C14, E52, E58.

Keywords: Nonlinear Taylor Rule, OECD forecasts, semi-parametric modeling, out-of-sample forecasting.

*The authors are particularly grateful to Stefan Sperlich, Charles Wyplosz, Henri Loubergé, Ulrich Kohli, Elvezio Ronchetti, Jean-Marc Natal and the colleagues from the Department of Economics as well as the participants of the Young Researchers Seminar of the University of Geneva for their valuable comments and insights.

†Address for correspondance: Nikolay Markov, Department of Economics, University of Geneva, Bvd. du Pont d'Arve 40, CH-1211 Geneva 4. E-mail: nicolay.markov@unige.ch

‡Address for correspondance: Carlos de Porres, Department of Economics, University of Geneva, Bvd. du Pont d'Arve 40, CH-1211 Geneva 4. E-mail: carlos.deporres@unige.ch

1 Introduction

"...It is appropriate to seek a formula that expresses the interest-rate operating target as a function of variables that can be observed (or at least estimated) by the central bank at the time that it must make its decision. Under this view, the Taylor rule does have certain basic features of the reaction functions associated with optimal policies. However, an optimal reaction function is unlikely to be quite so simple as the classic formulation of the Taylor rule."

Michael Woodford (2003)

In a seminal paper John Taylor has proposed in 1993 a monetary policy rule as a guideline for the Central Bank decisions on the key policy rate. This simple rule has proved to be robust across various specifications and different countries and has followed remarkably closely the actual pattern of the policy rates for most of the countries studied.[1] This paper builds on the previous literature to investigate more in-depth the theoretical and empirical specifications of the Taylor Rule. In particular, we derive a more flexible theoretical functional form of the Taylor Rule in order to empirically determine whether the policy rule is nonlinear. Indeed, the presence of nonlinearities in the policy reaction function could stem from a structural break in the specification, from asymmetric Central Bank preferences or from a nonlinear structure of the economy. In practice, the observed nonlinearity of the policy rule most likely reflects some combination of the three types previously mentioned.

One strand of the literature has investigated the presence of nonlinearities within a parametric modeling approach. In that spirit, the authors have performed either smoothed transition regressions or have estimated a Markov regime switching model. The former methodology has been followed by Alcidi et al. (2005) in their analysis of the U.S. monetary policy and more recently by Gerlach and Lewis (2010) for the case of the European monetary policy at the vicinity of the zero lower bound. The second approach has been applied by Owyang and Ramey (2004), Sims and Zha (2006) for the U.S. monetary policy and Assenmacher-Wesche (2005) for the case of the U.S., U.K. and Germany. Furthermore, Kim et al. (2005) have adopted the parametric methodology developed in Hamilton (2001) to show the presence of nonlinearities in the U.S. monetary policy. They have found that there is strong evidence for nonlinearity in the pre-Volcker period but not such evidence for the Volcker-Greenspan era within both backward and forward-looking Taylor Rules. Dolado et al. (2005) have analyzed the nonlinearity of the Taylor Rule for three European countries, the Euro area and the U.S. They have shown evidence for a nonlinear policy rule for all countries except for the U.S., stemming from a nonlinear Phillips curve but assuming a quadratic Central Bank loss function. Importantly, the nature of nonlinearity implies the inclusion of an interaction term between the inflation and the output gap expectations in the Taylor Rule specification. Within a parametric framework, the empirical evidence has shown that the latter interaction has proved to enter significantly the policy reaction functions of all the European Central Banks, but has been insignificant for the Fed. Cukierman and Muscatelli (2008) have investigated the nonlinearity of the Taylor Rule for the U.S. and U.K. assuming that the Central Bank has asymmetric preferences for inflation and the output gap and that the structure of the economy is linear. Using a parametric smooth transition regression methodology, the authors have found evidence in favor of nonlinearities in the U.K. and U.S. Taylor Rules, except during the Volcker era. More recently, Klose (2011) has shown evidence for the nonlinearity of the Taylor Rule for the European Central Bank (ECB) using a new approach based on the states of

[1] The reader could refer to the papers of Taylor and Williams (2010), Gorter, Jacobs and de Haan (2008), Clarida, Galí and Gertler (1998, 1999 and 2000) and Judd and Rudebusch (1998) for instance.

the economy. He has also found that the policy rule is nonlinear within each state when accounting for relevant quadratic terms of inflation and the output gap in the regressions.

A new strand of the literature investigates the nonlinearity of the Taylor Rule within a semi-parametric modeling approach. This type of estimator allows us to take into account more flexible specifications with respect to parametric ones. Therefore, it enables us to capture nonlinearities on the empirical support of the variables we are looking at, as well as to quantify the degree of smoothness between the dependent variable and the covariates. Along this line of research, Hayat and Mishra (2010), Conrad et al. (2010) apply a semi-parametric methodology to detect the presence of nonlinearities in the policy reaction function. The former have found evidence that the Fed reacts more strongly to inflation expectations that are in the range of 8-10%. In addition, for the forward-looking specification of the Taylor Rule the authors have found a significant effect from adding an interaction term of inflation and the output gap on the policy interest rate. Similarly, Conrad et al. (2010) provide evidence for the nonlinearity of the Taylor Rule for the European Central Bank (ECB) and the Fed as being driven mainly by the asymmetric preferences of the Central Bankers. The authors have shown that both Central Banks have reacted more aggressively to positive deviations of inflation expectations from their targets rather than to negative ones and that the degree of responsiveness has increased after some threshold levels are reached.

The goal of this paper is to challenge the linear specification of the Taylor Rule on the empirical ground based on a more general theoretical framework. It builds on the second strand of the literature and adopts a semi-parametric methodology to investigate the theoretical and empirical specifications of the policy reaction function. The paper contributes to the literature on the following aspects. First, we extend the empirical literature by investigating the functional form of the Taylor Rule for 8 OECD countries over the last three decades using OECD projections of economic fundamentals and Consensus Economics forecasts. Second, we propose a new semi-parametric specification of the Taylor Rule which provides a more accurate in-sample prediction of the policy stance in the countries investigated compared to the predictions stemming from a standard linear Taylor Rule. The performance of our favorite specification is close to the one of a Taylor Rule with policy inertia for most of the countries considered in the analysis. Importantly, it does not suffer from a possible misspecification problem often associated with the latter. Finally, the semi-parametric specifications systematically outperform the linear Taylor Rule for all countries within an out-of-sample forecasting of the policy rate. This result is robust to using two different forecasting procedures and two different measures of the economic outlook.

The structure of the paper is the following. Section 2 presents the theoretical framework, while section 3 discusses the data and the methodology that has been followed. Section 4 presents the results of the semi-parametric regressions and section 5 sheds some light on the out-of-sample forecasting performance of the models. Finally, section 6 offers some robustness analysis and section 7 presents the estimations with the consensus forecasts. The last section concludes on the main theoretical and empirical findings of the paper.

2 Theoretical framework

In this section we lay down the theoretical foundations characterizing a nonlinear economy in which a more flexible Taylor Rule, compared to the ones proposed so far in the literature

is derived.[2] The generalization comes from three main parts. Firstly, we assume that the IS curve is a possibly nonlinear smooth function in both arguments $E_t x_{t+1}$ and r_t, the expected future output gap and the real interest rate respectively.[3] Typically, in general equilibrium models we log-linearize the solution around the steady state in order to derive a closed form solution with respect to macroeconomic fundamentals. However, this procedure is only an approximation and remains valid only in a neighborhood of the steady state. Besides, the nonlinearity could capture other features of the economy as the presence of heterogeneous preferences for instance. In a seminal paper, Kirman (1992) criticizes the use of a representative agent in macroeconomics because it could yield misleading outcomes for collective behavior. The author states that the results from individual and aggregate utility maximization could be quite different, and in particular that the weak axiom of revealed preferences may not be satisfied at the aggregate level. Kirman also argues that one should consider individual heterogeneity and the interactions between agents over specific areas of the economy to more accurately model aggregate behavior. Secondly, in the same spirit, the New Keynesian Phillips Curve (NKPC) is also derived through log-linearization and, therefore, the smoothness could capture not only departures from the steady state but also some new features of the economy as the relevance of an interaction term between inflation and the output gap along the lines of Dolado et al. (2005). Finally, the Central Bank loss function is also assumed to be a more general smooth function whose third derivatives with respect to both π_t and x_t are assumed to exist. This could reflect an asymmetric responsiveness of the Central Bank regarding both the inflation and the output gap objectives. In departing from the standard quadratic loss function, the latter assumption relates the Central Bank behavior to the concept of prudence with respect to the economic fundamentals. In what follows, we present the theoretical framework, the main assumptions and the model derivation. Complete proofs of the propositions are reported in subsection 9.1 in the appendix. Consider the following New Keynesian model of the economy:

$$
\begin{aligned}
IS &: \quad x_t = \eta(E_t x_{t+1}, i_t - E_t \pi_{t+1}) + g_t && (1)\\
NKPC &: \quad \pi_t = \psi(E_t \pi_{t+1}, x_t) + u_t && (2)\\
L_t &= E_t \sum_{i=0}^{\infty} \beta^i h(x_{t+i}, \pi_{t+i}), \quad \beta < 1 && (3)
\end{aligned}
$$

where E_t denotes the expectation operator at time t, $\eta(E_t x_{t+1}, i_t - E_t \pi_{t+1})$ is a smooth function of the expected output gap and the real interest rate, $\psi(E_t \pi_{t+1}, x_t)$ is a smooth function of the expected inflation rate and the current output gap, L_t corresponds to the Central Bank loss function in period t and $h(x_{t+i}, \pi_{t+i})$ is a smooth function of the current and future output gaps and inflation rates. The stochastic processes for the demand-pull (g_t) and the cost-push (u_t) shocks are the following:

$$
\begin{aligned}
g_t &= \rho_g g_{t-1} + \epsilon_{g_t}, \mid \rho_g \mid < 1, \epsilon_{g_t} \sim iid(0, \sigma^2_{\epsilon_g}) && (4)\\
u_t &= \rho_u u_{t-1} + \epsilon_{u_t}, \mid \rho_u \mid < 1, \epsilon_{u_t} \sim iid(0, \sigma^2_{\epsilon_u}) && (5)
\end{aligned}
$$

We assume that the Central Bank minimizes the loss function (3) subject to the inflation equation (2) in each period. Then, from the first order conditions of the Central Bank optimization problem and from the model assumptions it can be shown that there exists

[2] For the theoretical specification see the influential work of Galí (2008), Woodford (2003) and Rotemberg and Woodford (1999).

[3] The output gap is defined as real GDP minus potential GDP expressed in percentage points of potential GDP.

an implicit relationship between the interest rate i_t and the economic fundamentals. The main theoretical assumptions and the derivation of the model are presented in the following subsections.

2.1 The model assumptions

Assumption A. *Let $h(x,\pi)$ be the Central Bank loss function with arguments x and π, the output gap and inflation respectively . Let us assume the following:*

1. $h_\pi(x,\pi) = \frac{\partial h(x,\pi)}{\partial \pi} < 0, \forall \pi < \bar{\pi}$ *and* $h_\pi(x,\pi) > 0, \forall \pi > \bar{\pi}$, *where* $\bar{\pi}$ *represents the inflation objective.*
2. $h_x(x,\pi) = \frac{\partial h(x,\pi)}{\partial x} < 0, \forall x < 0$ *and* $h_x(x,\pi) > 0, \forall x > 0$.
3. $h_{\pi\pi}(x,\pi) = \frac{\partial^2 h(x,\pi)}{\partial \pi^2} > 0, \forall \pi$ *and* $h_{xx}(x,\pi) = \frac{\partial^2 h(x,\pi)}{\partial x^2} > 0, \forall x$.
4. $h_{\pi x}(x,\pi) = \frac{\partial^2 h(x,\pi)}{\partial \pi \partial x} = h_{x\pi}(x,\pi) = 0$[4], $\forall \pi, x$
5. $h_{\pi\pi\pi}(x,\pi) = \frac{\partial^3 h(x,\pi)}{\partial \pi^3} \in \mathbb{R}$ *and* $h_{xxx}(x,\pi) = \frac{\partial^3 h(x,\pi)}{\partial x^3} \in \mathbb{R}$.

The assumption of null cross-derivatives is made essentially for model tractability. However, this hypothesis does not rule out any interaction between inflation and the output gap in the structure of the economy.[5]

Assumption B. *Let $\psi(E_t\pi_{t+1}, x_t)$ be the NKPC. Let us assume the following:*

1. $\psi_{E_t\pi_{t+1}}(E_t\pi_{t+1}, x_t) = \frac{\partial \psi(E_t\pi_{t+1}, x_t)}{\partial E_t\pi_{t+1}} > 0, \forall E_t\pi_{t+1}$ *and* t.
2. $\psi_{x_t}(E_t\pi_{t+1}, x_t) = \frac{\partial \psi(E_t\pi_{t+1}, x_t)}{\partial x_t} > 0, \forall x_t$ *and* t.
3. $\psi_{E_t^2\pi_{t+1}}(E_t\pi_{t+1}, x_t) = \frac{\partial^2 \psi(E_t\pi_{t+1}, x_t)}{\partial E_t^2\pi_{t+1}} \in \mathbb{R}, \forall t$ *and* $\psi_{x_t^2}(E_t\pi_{t+1}, x_t) = \frac{\partial^2 \psi(E_t\pi_{t+1}, x_t)}{\partial x_t^2} \in \mathbb{R}, \forall t$.
4. $\psi_{E_t\pi_{t+1}, x_t}(E_t\pi_{t+1}, x_t) = \frac{\partial^2 \psi(E_t\pi_{t+1}, x_t)}{\partial E_t\pi_{t+1} \partial x_t} = \psi_{x_t, E_t\pi_{t+1}}(E_t\pi_{t+1}, x_t) \in \mathbb{R}, \forall t$.

Assumption C. *Let $\eta(E_t x_{t+1}, i_t - E_t\pi_{t+1})$ be the IS curve. Let us assume the following:*

1. $\eta_{E_t x_{t+1}}(E_t x_{t+1}, i_t - E_t\pi_{t+1}) = \frac{\partial \eta(E_t x_{t+1}, i_t - E_t\pi_{t+1})}{\partial E_t x_{t+1}} > 0, \forall E_t x_{t+1}$ *and* t.
2. $\eta_{i_t - E_t\pi_{t+1}}(E_t x_{t+1}, i_t - E_t\pi_{t+1}) = \frac{\partial \eta(E_t x_{t+1}, i_t - E_t\pi_{t+1})}{\partial (i_t - E_t\pi_{t+1})} < 0, \forall i_t - E_t\pi_{t+1}$ *and* t.
3. $\eta_{E_t^2 x_{t+1}}(E_t x_{t+1}, i_t - E_t\pi_{t+1}) = \frac{\partial^2 \eta(E_t x_{t+1}, i_t - E_t\pi_{t+1})}{\partial E_t^2 x_{t+1}} \in \mathbb{R}, \forall t$ *and*
   $\eta_{(i_t - E_t\pi_{t+1})^2}(E_t x_{t+1}, i_t - E_t\pi_{t+1}) = \frac{\partial^2 \eta(E_t x_{t+1}, i_t - E_t\pi_{t+1})}{\partial (i_t - E_t\pi_{t+1})^2} \in \mathbb{R}, \forall t$.
4. $\eta_{E_t x_{t+1}, i_t - E_t\pi_{t+1}}(E_t x_{t+1}, i_t - E_t\pi_{t+1}) = \frac{\partial^2 \eta(E_t x_{t+1}, i_t - E_t\pi_{t+1})}{\partial E_t x_{t+1} \partial (i_t - E_t\pi_{t+1})} = \eta_{i_t - E_t\pi_{t+1}, E_t x_{t+1}}(E_t x_{t+1}, i_t - E_t\pi_{t+1}) \in \mathbb{R}, \forall t$.

[4] The absence of interaction implies additivity in the Central Bank loss function:

$$h(x_t, \pi_t) = h_1(x_t) + h_2(\pi_t) \tag{3.1}$$

[5] Cukierman and Muscatelli (2008) further assume that $h_{\pi\pi\pi}(.) \geq 0$ and $h_{xxx}(.) \leq 0$. However, our semi-parametric approach permits to remain more general as regards this assumption.

2.2 The model derivation

Proposition 1. *Let Assumptions A, B and C hold. Furthermore, assume the Central Bank solves the following optimization problem in each period t:*

$$\begin{aligned} & \min_{x_t,\pi_t} E_t h(x_t,\pi_t) \\ & s.t. \\ \pi_t &= \psi(E_t\pi_{t+1},x_t)+u_t \end{aligned} \tag{6}$$

Then, it follows that:

1. *The first order condition (FOC) is given by :*

$$\begin{aligned} &E_t[h_x(\eta(E_tx_{t+1},i_t-E_t\pi_{t+1})+g_t,\psi(E_t\pi_{t+1},x_t)+u_t)+ \\ &h_\pi(\eta(E_tx_{t+1},i_t-E_t\pi_{t+1})+g_t,\psi(E_t\pi_{t+1},x_t)+u_t)\psi_x(E_t\pi_{t+1},x_t)]=0 \end{aligned}$$

2. *By totally differentiating the FOC with respect to E_tx_t and $E_t\pi_t$ and setting $t=0$, we obtain:*

$$\begin{aligned} &E_0[h_{xx}(x_0,\pi_0)+h_\pi(x_0,\pi_0)\psi_{xx}(E_0\pi_1,x_0)]dE_0x_0+ \\ &+E_0[h_{\pi\pi}(x_0,\pi_0)\psi_x(E_0\pi_1,x_0)]dE_0\pi_0=0 \end{aligned} \tag{7}$$

3. *Finally, from the FOC and the model assumptions there exists an implicit relationship between the interest rate i_t and the economic fundamentals. The latter defines a nonlinear Taylor Rule of the following form:*

$$i_t=\Omega(E_t\pi_{t+1},E_tx_{t+1}) \tag{8}$$

It is of great interest to derive the properties of the nonlinear Taylor Rule with respect to $E_t\pi_{t+1}$ and E_tx_{t+1}. These results shall describe the behavior of the above defined policy rule regarding macroeconomic fundamentals.

Proposition 2. *Let i_t be defined by $\Omega(E_t\pi_{t+1},E_tx_{t+1})$. Then, it follows that:*

1. $$\frac{di_0}{dE_0\pi_1}=1+\frac{-E_0[h_{\pi\pi}(.)\psi_x(.)\psi_{E_0\pi_1}(.)]}{\underbrace{E_0\left\{[h_{xx}(.)+h_\pi(.)\psi_{xx}(.)]\eta_{i_0-E_0\pi_1}(.)+h_{\pi\pi}(.)\psi_x^2(.)\eta_{i_0-E_0\pi_1}(.)\right\}}_{=A}} \tag{9}$$

   where the Taylor Principle is verified as long as we assume that $h_\pi(.)\psi_{xx}(.)\geq 0$. In the standard linear framework this term is zero which implies that the Taylor Principle holds. However, in this more general setting the Taylor Principle might not hold over some of the support of the variables.

2. $$\frac{di_0}{dE_0x_1}=-\frac{\eta_{E_0x_1}(.)}{\eta_{i_0-E_0\pi_1}(.)}>0 \tag{10}$$

3.

$$\frac{d^2 i_0}{d\left(E_0\pi_1\right)^2} = -\frac{\alpha_1\alpha_2 - \alpha_3\left(\alpha_4+\alpha_5\right)}{\alpha_6} \tag{11}$$

where

$$\begin{aligned}
\alpha_1 &\equiv E_0[h_{\pi\pi\pi}(.)\psi^2_{E_0\pi_1}(.)\psi_x(.) + h_{\pi\pi}(.)\psi_{xE_0\pi_1}(.)\psi_{E_0\pi_1}(.) + h_{\pi\pi}(.)\psi_x(.)\psi_{E_0^2\pi_1}(.)] \\
\alpha_2 &\equiv E_0\{[h_{xx}(.) + h_\pi(.)\psi_{xx}(.) + h_{\pi\pi}(.)\psi^2_x(.)]\eta_{i_0-E_0\pi_1}(.)\} \\
\alpha_3 &\equiv E_0[h_{\pi\pi}(.)\psi_x(.)\psi_{E_0\pi_1}(.)] \\
\alpha_4 &\equiv E_0\{[h_{\pi\pi}(.)\psi_{E_0\pi_1}(.)\psi_{xx}(.) + h_\pi(.)\psi_{xxE_0\pi_1}(.) - h_{xxx}(.)\eta_{E_0\pi_1}(.)]\eta_{i_0-E_0\pi_1}(.)\} \\
&\quad -E_0\{[h_{xx}(.) + h_\pi(.)\psi_{xx}(.)]\eta_{(i_0-E_0\pi_1)E_0\pi_1}(.)\} \\
\alpha_5 &\equiv E_0\{[h_{\pi\pi\pi}(.)\psi_{E_0\pi_1}(.)\psi_x(.)^2 + 2h_{\pi\pi}(.)\psi_{xE_0\pi_1}(.)]\eta_{i_0-E_0\pi_1}(.)\} - E_0[h_{\pi\pi}(.)\psi^2_x(.)\eta_{(i_0-E_0\pi_1)E_0\pi_1}(.)] \\
\alpha_6 &\equiv \left(E_0\{[h_{xx}(.) + h_\pi(.)\psi_{xx}(.) + h_{\pi\pi}(.)\psi^2_x(.)]\eta_{i_0-E_0\pi_1}(.)\}\right)^2
\end{aligned}$$

4.

$$\frac{d^2 i_0}{d\left(E_0 x_1\right)^2} = -\frac{\eta_{E_0^2 x_1}(.)\eta_{i_0-E_0\pi_1}(.) - \eta_{E_0 x_1}(.)\eta_{i_0-E_0\pi_1,E_0 x_1}(.)}{\left(\eta_{i_0-E_0\pi_1}(.)\right)^2} \tag{12}$$

5.

$$\frac{d^2 i_0}{dE_0 x_1 dE_0\pi_1} = \frac{\eta_{E_0 x_1,E_0\pi_1}(.)\eta_{i_0-E_0\pi_1}(.) - \eta_{E_0 x_1}(.)\eta_{i_0-E_0\pi_1,E_0\pi_1}(.)}{\left(\eta_{i_0-E_0\pi_1}(.)\right)^2} \tag{13}$$

$$\begin{aligned}
\frac{d^2 i_0}{dE_0\pi_1 dE_0 x_1} &= \frac{E_0[h_{\pi\pi}(.)\psi_x(.)\psi_{E_0\pi_1}(.)]E_0[h_{xxx}(.)\eta_{E_0 x_1}(.)\eta_{i_0-E_0\pi_1}(.) + (h_{xx}(.) + h_\pi(.)\psi_{xx}(.))}{\left(E_0[(h_{xx}(.) + h_\pi(.)\psi_{xx}(.) + h_{\pi\pi}(.)\psi_x(.)^2)\eta_{i_0-E_0\pi_1}(.)]\right)^2} \\
&\quad \frac{\eta_{i_0-E_0\pi_1,E_0 x_1}(.)] + E_0[h_{\pi\pi}(.)\psi_x(.)\psi_{E_0\pi_1}(.)]E_0[h_{\pi\pi}(.)\psi_x(.)^2\eta_{i_0-E_0\pi_1,E_0 x_1}(.)]}{\left(E_0[(h_{xx}(.) + h_\pi(.)\psi_{xx}(.) + h_{\pi\pi}(.)\psi_x(.)^2)\eta_{i_0-E_0\pi_1}(.)]\right)^2}
\end{aligned}$$

Remark: *Note that the derivative of the Taylor Rule with respect to the expected output gap should be positive on the entire support of the variable. However, this is not always the case empirically as the evidence in some countries points it out. This fact is in line with both the linear theoretical framework and empirical results in the literature.*

It is important to highlight that in the case the Central Bank loss function depends on the change in the output gap, as outlined in Clarida, Galí and Gertler (1999) and in Walsh (2003), the derivation of the Taylor Rule becomes particularly cumbersome and therefore the analysis is performed empirically in the following sections. At this stage we do not make any further assumptions neither on the behavior of the Central Bank nor on the structure of the economy. The study of the functional form of the Taylor Rule is thus left for the empirical part of the paper. However, we would like to stress out that our semi-parametric approach is particularly flexible in modeling any form of nonlinearities in the monetary policy rule.[6] In fact, this framework is well suited for capturing any change in the Central Bank's responsiveness that could stem either from a change in the preferences of the monetary authorities, from a change in the structure of the economy or/and from a structural break. The first two types of nonlinearities are reflected in the curvature of the Taylor Rule on the support of the series. The latter refers to a change in the policy response for a given value of the variables.

[6] This methodology assumes that the relationship between the dependent variable and the covariates is at least twice differentiable with respect to the latter.

3 Data and methodology

We have built-up a data set that contains quarterly OECD forecasts of inflation and real GDP growth for 8 industrialized countries: Australia, Canada, Japan, New Zealand, Norway, Sweden, Switzerland and USA. All data have been collected from Datastream and the official Central Banks' websites. The inflation and real output growth forecasts refer to a one-year horizon for the current year with respect to the corresponding quarter of the previous year. An important feature of the data is that we have created a comprehensive database that spans the period from 1976 Q3 to 2010 Q4 for all the countries previously mentioned. These data enable us to closely track and accurately evaluate the actual monetary policy stance that has been implemented by the 8 Central Banks over the relatively long period considered in the analysis. Besides, the OECD database ensures a proper comparability of the estimation results between the countries investigated.[7] In all specifications the dependent variable refers either to the key policy interest rate or to the closest market interest rate targeted by the Central Bank depending on data availability. In addition, the policy rate used in the regressions accounts for the fact that the definition of the key interest rate has changed over time for several Central Banks. In section 7, we also present the estimation results with Consensus Economics monthly forecasts for Canada, Japan, Norway, Sweden, Switzerland and USA over the period December 1989-March 2010. The advantage of using the consensus data is that the reported forecasts are forward-looking and are provided in real-time. In the regressions we use the forecasts of inflation and real GDP growth with a fixed horizon of one-year.

The appendix provides a detailed description of the variables used in the empirical analysis and table 1 displays some summary statistics. The table shows that the policy rate has been high in the countries that have experienced important levels of inflation. New Zealand is the country with the highest average inflation rate while Japan has the lowest. The GDP growth forecasts are quite similar among the OECD countries with Australia scoring the highest average growth rate and Switzerland featuring the lowest. The forecasts for the change in the output gap exhibit a smaller variability compared to the GDP growth projections and are on average close to zero for all countries. We have also performed some unit root and stationarity tests which are reported in table 2. The latter point out that the policy rate and the inflation forecasts are likely to be nonstationary in most of the countries even though the Augmented Dickey-Fuller statistics often show evidence against the null hypothesis of unit root. However, the nonstationarity of these series is not problematic because our estimations are performed within the support of the variables and not directly in the time series dimension. There is also a strong evidence against the null hypothesis of unit root for the real GDP growth and output gap change forecasts in all countries. Table 3 presents some summary statistics of the consensus data and table 4 displays some unit root and stationarity tests. The latter show that for most of the countries there is no strong evidence against the null hypothesis of unit root for the policy rate and the inflation forecasts while the output growth and the output gap change forecasts are clearly stationary. The Augmented Dickey-Fuller tests point out that the series are stationary.

Given that the aim of this chapter is to analyze the responsiveness of the Central Bank to economic fundamentals using the available relevant information we do not consider the problem of estimating the level of potential output. Hence, we use forecasts of real output growth instead of the output gap in order to avoid the mismeasurement problems of potential output encountered in actual policy making along the lines of Kai and Lonning (2006) for instance. However, in the robustness section we also comment on an analysis

[7] The OECD reports its most accurate projections of economic fundamentals which permits to model more truly the Central Bank reaction function for each country.

performed with the Hodrick-Prescott (H-P) detrended output growth rate also known as a speed limit policy. Indeed, Walsh (2003) has shown that under discretion a Central Bank that targets the difference between the output growth rate and the growth rate of potential output (the change in the output gap) will deliver the optimal pre-commitment policy outcome. Walsh has emphasized that the implementation of a speed limit policy is particularly relevant in the presence of measurement errors in potential output and could be welfare enhancing compared to pure discretion or to an inflation targeting regime. We first present the analysis with the output growth forecasts and then perform the regressions with the H-P detrended change in the output gap in the robustness section. This approach also permits to use more accurate data in the estimations and to avoid the potential end-of-period problems that arise when applying the H-P filter. The method is also in line with the approach of Taylor (1993) as well as with a recent estimation of a Taylor Rule for the Fed performed in Coibion and Gorodnichenko (2011).

As regards the methodology, we use a novel approach in the estimation procedure. In order to account for potential nonlinearities in the Taylor Rule specification we have adopted a semi-parametric modeling technique.[8] Based on the approach of Hastie and Tibshirani (1986 and 1990) we specify an additive model which is a linear model that contains smooth functions of covariates.[9] This specification is quite flexible in capturing any potential nonlinearities in the Taylor Rule, while features the advantage of avoiding the curse of dimensionality problem. The GAM specifications are all estimated in R using penalized regression splines. The estimation is performed in two stages. First, the degree of smoothness of the splines is estimated using a Generalized Cross Validation (GCV) algorithm. Even though this procedure penalizes model overfitting, we implement the GCV method using a $\gamma = 1.4$. The latter is an additional penalty term that strikes the right balance between matching as closely as possible the data and providing accurate out-of-sample forecasts as suggested by Kim and Gu (2004). Second, the smooth functions are estimated with Penalized Iterative Reweighted Least Squares (P-IRLS) as outlined in Wood (2006). The GAM approach is also preferred because the methodology relies on orthogonal bases unlike kernel based estimators[10] (Nadaraya-Watson for instance). The following section presents the empirical results of the estimations.

4 Empirical evidence

In this section we present the empirical results of the estimation of the Taylor Rule for the eight OECD Central Banks using the GAM approach presented above. Three main specifications of the Taylor Rule have been investigated: a bivariate GAM, a univariate GAM without interaction terms and a univariate GAM with interaction terms. We first start by presenting the estimation results for the first model in the following subsection. Tables 5, 6 and 7 in the appendix report the results from all the regressions performed in this section.

4.1 Bivariate GAM

In this subsection we have estimated the following GAM specification for each country in the data set:

[8] An extensive treatment of the non-parametric and semi-parametric methods is provided in Li and Racine (2007) and in Wood (2006).

[9] The adopted methodology closely follows Woods (2006). This additive model is a special case of a general family of semi-parametric models known as Generalized Additive Models (GAM). The additive specification is obtained from the GAM using the canonical link due to the continuity of the dependent variable.

[10] Orthogonal series estimators can easily handle interaction terms.

$$i_t = c + s(E_t\pi_{t+1}, E_t x_{t+1}) + \epsilon_t \tag{14}$$

where i_t is the policy interest rate, c is a parametric constant term, $E_t\pi_{t+1}$ and $E_t x_{t+1}$ are the inflation and real GDP growth forecasts for the current year respectively. $s(E_t\pi_{t+1}, E_t x_{t+1})$ is a smooth function of the explanatory variables to be estimated and ϵ_t is a stochastic disturbance term.

The main purpose of this estimation is to show the presence of nonlinearities in the standard specification of the Taylor Rule.[11] Figures 9 to 16 in the appendix present the perspective plots of the estimated smooth functions. The latter show clear evidence for nonlinearity in the policy reaction functions.[12] The degree of nonlinearity is measured by the estimated degrees of freedom of the smooth functions.[13] More precisely, a linear Taylor Rule would imply an estimated degree of freedom of one, while a higher value of the latter points to an increasing degree of nonlinearity. In that perspective, it is interesting to notice that the degree of nonlinearity in the Taylor Rule varies considerably among the countries considered in the regressions. Indeed, the country with the highest degree of nonlinearity is found to be Sweden with an estimated degree of freedom of the smooth function $s(E_t\pi_{t+1}, E_t x_{t+1}) = 21.84$, while the lowest degree of nonlinearity in the Taylor Rule is reported for Switzerland with an estimated degree of freedom of the smooth term $s(E_t\pi_{t+1}, E_t x_{t+1}) = 4.39$. All the estimated smooth functions are highly statistically significant. In addition, the model performs well since the explained deviance is relatively high for most of the countries. The highest explained deviance (0.844) is obtained for Japan, while the lowest explained deviance of the model (0.439) is reported for Norway.

The figures of the bivariate GAM thus bring evidence in favor of a nonlinear specification of the Taylor Rule that is robust across all the eight OECD Central Banks considered in the regressions. Furthermore, the perspective plots show that it might be important to account for a possible interaction between the inflation and output growth forecasts in order to accurately predict the appropriate path of the key policy rate in each country. This fact can be explained by the presence of non-zero cross-derivatives between the interest rate and macroeconomic fundamentals.

In the following subsection we include the inflation and real GDP growth forecasts additively in the GAM specification in order to get a closer interpretation of the estimation results to the ones of the standard Taylor Rule (1993). We show that even the closest analog to the original Taylor Rule brings evidence in favor of a nonlinear specification of the policy reaction function for most of the countries.

4.2 Univariate GAM without interaction terms

In this part of the paper we estimate the standard Taylor Rule specification within the above semi-parametric framework in which the explanatory variables enter additively the GAM. The estimated specification takes the following form:

$$i_t = c + s_1(E_t\pi_{t+1}) + s_2(E_t x_{t+1}) + \epsilon_t \tag{15}$$

which differs from (14) in that the smooth functions of the explanatory variables are estimated additively. This specification provides a more convenient interpretation of the

[11] Even though the econometric procedure provides consistent estimates of the smooth functions, it is not the most appropriate specification given the number of observations used in the estimations. The latter is due to a lower rate of convergence of the bivariate GAM, $n^{-\frac{2}{3}}$, compared to either a univariate GAM, $n^{-\frac{4}{5}}$ or a parametric model, n^{-1}. Therefore, we include the covariates additively in the following univariate specifications.

[12] As clear evidence for nonlinearity we refer to the estimated large degrees of freedom of the smooth term.

[13] The degrees of freedom correspond to the number of parameters needed to estimate the smooth terms.

Central Bank behavior in terms of its inflation and real output growth responsiveness. The estimation results are reported in table 6 in the appendix. In what follows we comment in details the estimation results for each Central Bank in our sample.

4.2.1 Australia

The Reserve Bank Act of 1959 assigns three explicit policy objectives to the Reserve Bank of Australia (RBA): the stability of the national currency, the maintenance of full employment and the contribution to the economic prosperity and welfare of the people in Australia, the latter being achieved mainly through the first two goals. Since the adoption of an inflation targeting policy the Central Bank has focused on maintaining price stability in the medium term as an overriding monetary policy goal. The latter strategy has thus provided a flexible institutional framework that has fostered the price stability commitment of the RBA. The Statement on the conduct of monetary policy between the Governor and the Treasurer defines the inflation target as an average inflation rate between 2% and 3% in the medium term (over the cycle) without specifying it in terms of an inflation band or as a midpoint target level. The short term overnight lending rate (the cash rate) has been used as the key policy rate. We first start the analysis of the Australian monetary policy by presenting the estimates of the smooth functions of inflation and real GDP growth forecasts in figure 1 below.[14]

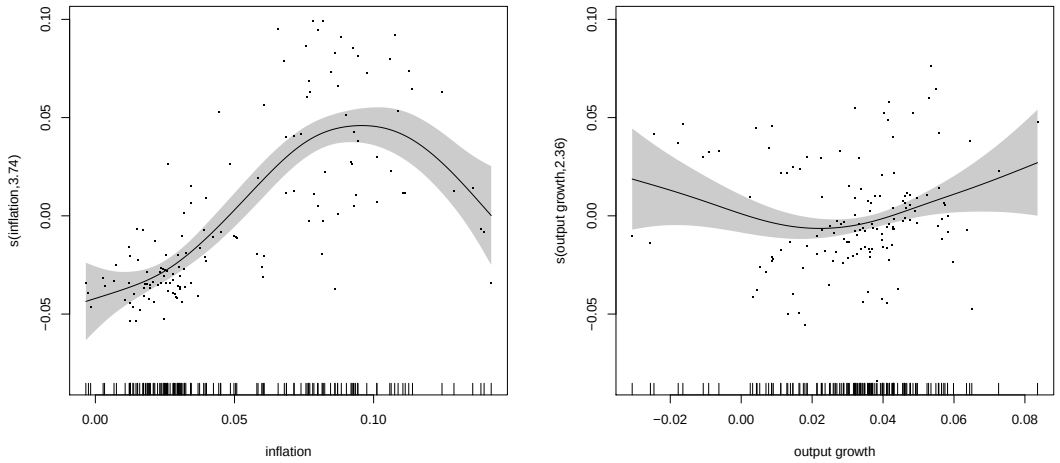


Figure 1: Estimated smooth functions for Australia

First, it is important to emphasize that the estimated smooth functions are statistically significant and the model features good explanatory power given that the explained deviance is about 64.1%. At a first look at the graphs, one can notice that both the inflation and output growth responsiveness of the RBA are clearly nonlinear as indicated by the estimated degrees of freedom and the large swings in the smooth functions. The latter score 3.74 and 2.36 for the smooth functions of inflation and the output growth forecasts respectively. Hence, this evidence suggests that the reaction of the Central Bank to economic fundamentals has changed along the levels of the inflation and output growth forecasts.

[14]On each figure the shaded area displays a 95% confidence interval for the estimated smooth terms and the dots correspond to the partial residuals. The reported covariates are intended for the forecasts of inflation and real output growth rates.

As regards the reaction to the inflation projections, the RBA's responsiveness is strongly increasing in the levels of the forecasts ranging from around 0% to close to 10%. Thereby, it seems that for these levels of inflation the Central Bank is worried about any potential threat to its price stability commitment. Then, for inflation forecasts that are above that threshold the responsiveness changes and swiftly decreases with the level of the forecasts. This reversal of the policy behavior might point out that once a certain inflation threshold is reached the Central Bank is actually more concerned about pursuing other policy goals such as stabilizing real output growth rather than inflation expectations. As inflation expectations are not anchored with the price stability objective of the RBA, the latter suffers from an eroded inflation aversion credibility. This in turn reflects an inflation destabilizing policy which could lay the ground for self-fulfilling bursts of inflation.

Concerning the reaction to real output growth, the Central Bank features an asymmetric policy responsiveness around a threshold level for the real output growth forecasts about 2%. Indeed, for output growth forecasts ranging from around -3% to close to 2% the RBA's reaction is decreasing along the level of the forecasts thus implying a destabilizing policy for the economic outlook. Conversely, for output growth projections above 2% its responsiveness increases along the level of the forecasts thus exerting a stabilizing effect on the economy. Hence, the threshold level of 2% might be in line with the growth level of potential output in Australia. Thereof, as the growth rate of real output exceeds the growth rate of the potential, the Central Bank implements a speed limit policy in the words of Walsh (2003) by increasing swiftly the policy rate to prevent the build-up of inflationary pressures in the economy that could threaten the pursuit of price stability as its ultimate monetary policy goal.

Finally, a more careful inspection of the previous figure reveals that the Taylor Principle is satisfied for moderate levels of the inflation expectations and does not hold over the entire support of the inflation forecasts.[15] In particular, the latter is verified for inflation expectations that are in the range of 3% to 8%. Any higher inflation rates bring evidence that monetary policy has not exerted a stabilizing effect on inflation as the Central Bank might have focused on pursuing other policy goals as previously mentioned.

4.2.2 Canada

Monetary policy in Canada is implemented within a flexible inflation targeting framework similarly to the policy strategy in Australia. The Bank of Canada has specified a target range for inflation between 1% and 3% aiming to achieve the midpoint of 2% over the medium term. The Central Bank has also defined an operating target for the uncollateralized overnight market lending rate as its main policy interest rate. Figure 2 displayed below presents the estimated smooth functions of the inflation and output growth forecasts.

At a first glance figure 2 reveals a clearly nonlinear behavior of the Central Bank with respect to economic fundamentals. Indeed, the estimated degrees of freedom of the inflation smooth function is 3.56, while the smooth function of the output growth forecasts scores an estimated degree of freedom of 4.51. Besides, the estimated smooth functions are all highly statistically significant and the model performs well since it explains about 77.1% of the model's deviance.

The Bank of Canada's responsiveness to the inflation forecasts is clearly nonlinear as reflected in the shape of the estimated smooth function. Globally, the Central Bank has increased the policy rate in the event of a rise in the inflation expectations over the entire support. However, the degree of responsiveness changes along the forecasted values. Indeed, for any forecasts between 2% and 5% or above 9% the Central Bank strongly

[15] The Taylor Principle can be checked by drawing a straight line with a slope of 1 on the corresponding graph and comparing it to the slope of the inflation smooth function. It is verified if the reaction of the Central Bank is more than proportional to changes in inflation expectations.

increases the policy rate in the event of a rise in the inflation projections. For the remaining range of the forecasts, its inflation responsiveness is much softer suggesting that the Bank of Canada might have prioritized other policy goals or has reacted to some unexpected shocks. Contrarily to the RBA, the Bank of Canada has responded quite aggressively to high inflation forecasts by sharply raising the key policy rate.

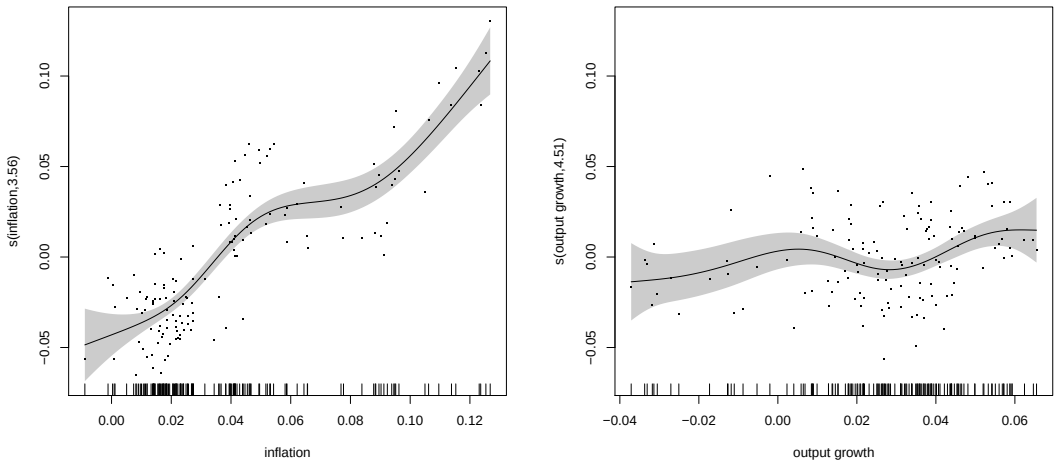


Figure 2: Estimated smooth functions for Canada

As regards the reaction to the economic outlook, the Central Bank's policy has featured a nonlinear policy pattern as well. Indeed, the estimated smooth function shows that the policy responsiveness is likely to be asymmetric around a threshold level of close to 3%. Hence, for any output growth forecasts between 0% and 3% the Bank of Canada exerts a destabilizing effect on the economic outlook by lowering the policy rate in the event of a rise in the growth forecasts. Conversely, for any real output growth forecasts above 3% the Central Bank is likely to increase the policy rate in the event of a rise in the forecasts thus exerting a stabilizing behavior on the economic outlook. The evidence for an asymmetric policy responsiveness of the Bank of Canada suggests that the Central Bank might have assigned different weights on the stabilization of inflation relative to the economic outlook depending on the level of the fundamentals. Nonetheless, it is possible that the shape of the smooth functions reflects some nonlinearity in the structure of the economy.

Concerning the Taylor Principle, the latter is satisfied only within some part of the support of the inflation expectations, particularly for some moderate levels of the forecasts that are between 2% and 5%, as well as for any projections above 9% for instance.

4.2.3 Japan

The monetary policy framework of the Bank of Japan (BoJ) is based on an explicit definition of price stability as the overriding goal : the Central Bank is "... *aimed at achieving price stability, thereby contributing to the sound development of the national economy*". However, the BoJ has not assigned any particular numerical value to the definition of price stability in light of the recent experience with the price developments in Japan. Indeed, according to the BoJ the latter would not be consistent with the long term growth potential of the economy. In order to achieve its goal the Central Bank sets an operating target for the uncollateralized overnight market interest rate (the call rate). However, the monetary policy strategy does not feature any explicit policy framework, contrarily to the inflation targeting Central Banks previously presented. Figure 3 shown below displays the

estimated smooth functions of inflation and real output growth for the BoJ.

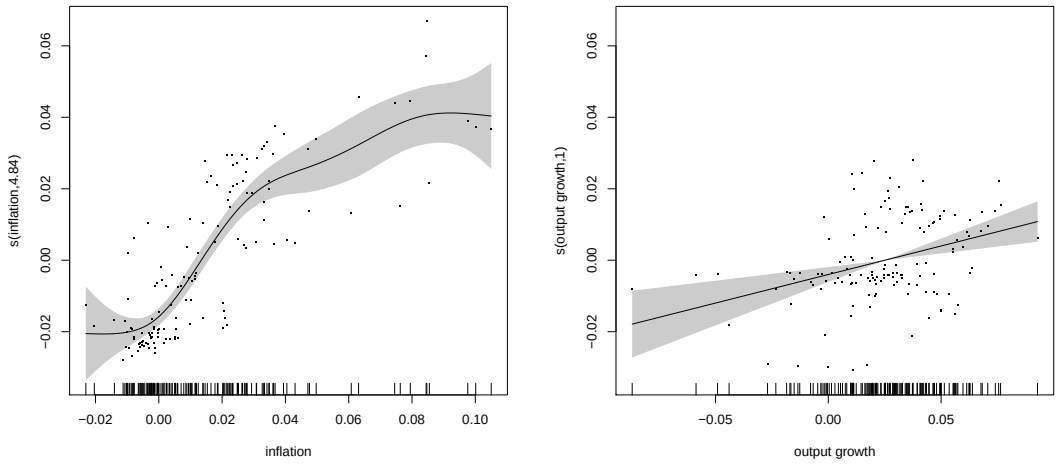


Figure 3: Estimated smooth functions for Japan

The empirical evidence reveals that the inflation responsiveness of the BoJ has been clearly nonlinear, while it has responded linearly to the real output growth forecasts. Indeed, the estimated degrees of freedom of the smooth functions of inflation and output growth forecasts are 4.84 and 1 respectively. Both the estimated inflation and real output growth smooth functions are highly statistically significant and the model performs well featuring an explained deviance of about 77.9%.

As regards the reaction to inflation expectations, the BoJ's responsiveness is swiftly increasing for any moderate levels of the inflation forecasts. Thereby, the BoJ strongly reacts to rises in the inflation forecasts that are between -1% and 3%. Then, for any projections above the upper level its inflation responsiveness still expands but at a much slower pace. The behavior of the Central Bank has been consistent with its mandate of preserving price stability since the BoJ has offset any increases in inflationary expectations by raising the policy rate.

However, a closer look at the above figure points out that the Taylor Principle might not have been satisfied over some of the range of the forecasts. The Central Bank has reacted the most aggressively to the inflation projections that are between 0% and 3%, while it has not sufficiently adjusted the policy rate out of this range in order to exert a stabilizing effect on inflation expectations. This evidence points out that the Japanese Central Bank could benefit from the adoption of an explicit monetary policy framework that will help to credibilize the price stability commitment of the monetary authorities in order to anchor the inflation expectations with the policy objective.

Figure 3 also reveals a linear policy response of the BoJ with respect to the output growth forecasts, a result which stands in contrast with the findings for the preceding Central Banks. The estimated smooth function indicates an increasing policy responsiveness of the BoJ along the support of the output growth forecasts implying a stabilizing effect on the economic outlook. The latter is thus consistent with the monetary policy mandate. An interesting result from the model's fit is that the latter predicts negative nominal policy interest rates for periods of very low and sluggish output growth and deflationary expectations. Indeed, Japan has experienced a prolonged period of a particularly sluggish economic growth in the 1990's and during the recent decade, which might account for the very low level of the policy rate. Thereof, the estimated model captures pretty well the Zero Lower Bound (ZLB) problem that the Japanese monetary authorities have been

facing during most of the last two decades.

4.2.4 New Zealand

The Reserve Bank of New Zealand (RBNZ) has been at the forefront of inflation targeting since it is the first Central Bank to adopt this strategy as an explicit monetary policy framework. This transparent policy design has fostered the price stability commitment of the Central Bank and has been successful in anchoring the private sector inflation expectations with its price stability objective. The Policy Targets Agreement (PTA) between the RBNZ's Governor and the Minister of Finance defines the inflation target as an average rate of inflation between 1% and 3% that has to be achieved in the medium term. Moreover, the Central Bank Act stipulates that "*...in pursuing its price stability objective, the Bank shall implement monetary policy in a sustainable, consistent and transparent manner and shall seek to avoid unnecessary instability in output, interest rates and the exchange rate*". In order to meet the policy objectives the RBNZ has specified an operating target for the short term overnight market lending rate (the cash rate).

On the one hand, the figure displayed below points to a linear and increasing responsiveness of the RBNZ to the economic outlook, even though the estimated smooth function appears to be not statistically significant. On the other hand, the reaction to inflation expectations has been clearly nonlinear and highly statistically significant. The estimated smooth function scores 3.22 estimated degrees of freedom. As regards the goodness of fit, the model's explained deviance amounts to 56.6%.

Concerning the reaction to the inflation expectations, the RBNZ features an increasing but changing policy responsiveness along the level of the forecasts. Indeed, it is strongly increasing for moderate levels of the forecasts, while it augments at a slower pace for higher inflation projections. This evidence points out that the Central Bank might have assigned different weights to the inflation and output growth stabilization goals depending on the state of the economy.

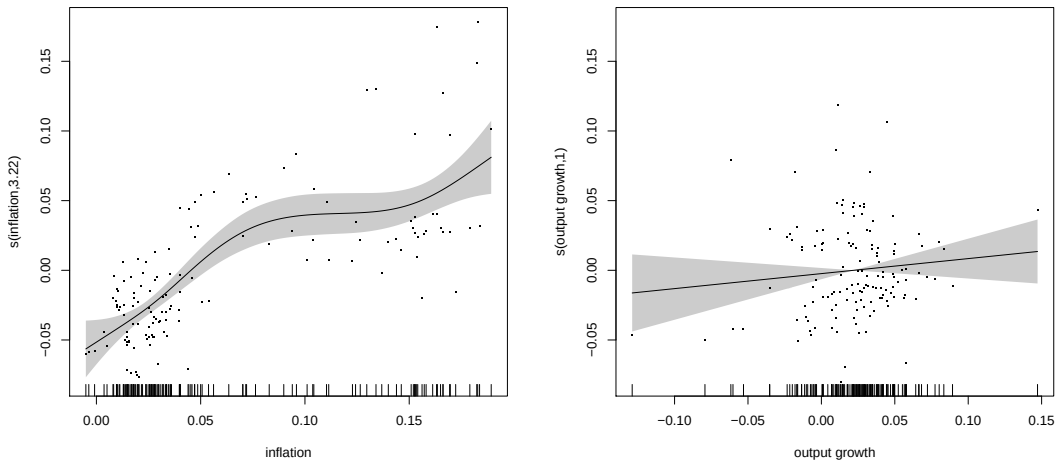


Figure 4: Estimated smooth functions for New Zealand

The reaction of the RBNZ to the output growth expectations is very much alike the spirit of the BoJ. Indeed, the latter is linear and increasing in the levels of the forecasts implying a stabilizing policy effect on the economic outlook. Hence, this behavior is in line with the monetary policy mandate. As regards the Taylor Principle, the Central Bank raises the policy rate more than proportionally with any inflation forecasts which

are located between 0% and 8%. For higher inflation forecasts, the Taylor Principle is not satisfied. In fact, since the Central Bank has not secured price stability for these high forecast levels, it may have favored the emergence of self-fulfilling bursts of inflation expectations.

4.2.5 Norway

The Bank of Norway implements monetary policy within a flexible inflation targeting framework. The government has defined an operational target for inflation that should be close to 2.5% over the medium term. In addition, monetary policy should also contribute to stabilizing output and employment. The Central Bank sets as a policy instrument the overnight rate on banks' deposits.[16] Figure 5 displayed below presents the estimated smooth functions of the inflation and output growth forecasts.

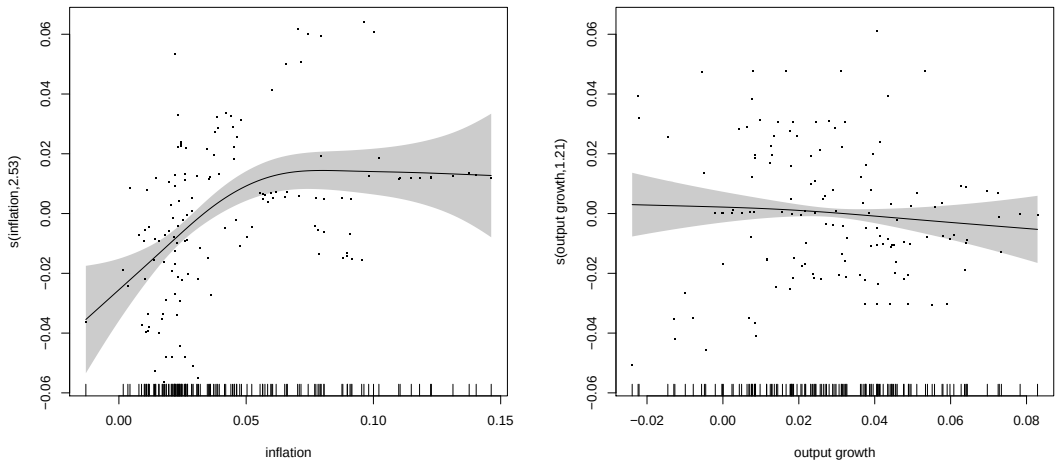


Figure 5: Estimated smooth functions for Norway

At a first look, the inflation smooth function points to a significant and nonlinear policy response of the Bank of Norway with an estimated degree of freedom of 2.53. As regards the reaction to output growth, the latter is close to linear and not statistically significant. Indeed, the estimated degrees of freedom amount to 1.21. Furthermore, the model's fit is the lowest compared to the other Central Banks featuring an explained deviance of about 25.5%.

The inflation responsiveness function reveals that the Central Bank's reaction is increasing for any inflation expectations located in the -2% to around 7% interval. Then, for higher forecasts the reaction of the Bank of Norway to inflation flattens sharply implying a destabilizing effect on inflation expectations. This evidence shows that the Central Bank's behavior changes along the level of the fundamentals as the Bank of Norway might have pursued other policy goals or might have responded to unexpected shocks stemming from the structure of the economy.

As regards the reaction to the output growth projections, the latter is nearly non existent for any forecasts up to 2% and then is slightly decreasing for higher forecast levels. Thus, the Bank of Norway's policy has been accommodating both for high levels of

[16] It is important to emphasize that the Bank of Norway has set a target for the sight deposit rate as its key policy rate. However, given that the overnight lending rate is the prevalent rate that Central Banks seek to influence, we have decided to use the latter in the empirical analysis. This ensures a proper comparability of the results with those obtained for the other Central Banks.

inflation and real output growth forecasts. This evidence points out that the Central Bank could have focused more on pursuing other policy goals such as stabilizing the Norwegian crown, which might have played an important role in the transmission of monetary policy impulses to the economy.

The Central Bank has been very close to satisfying the Taylor Principle for very low levels of the inflation forecasts, even though one can state that in general the Bank of Norway has not sufficiently raised its policy rate to exert a stabilizing effect on inflation expectations.

4.2.6 Sweden

The Swedish Riksbank implements monetary policy within a formal inflation targeting framework. The Central Bank Act does not assign other policy objectives to the Riksbank except to promote a safe and efficient payments system. The operational objective for inflation has been specified as a point target of 2%. The Riksbank has set the repo rate as an operating target for the one-week interbank market lending rate. The estimated smooth functions are displayed in figure 6 below.

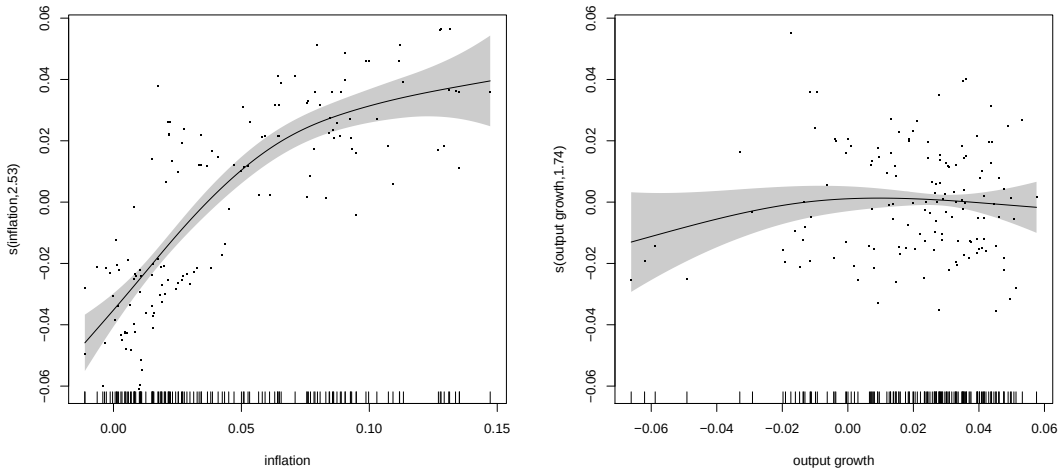


Figure 6: Estimated smooth functions for Sweden

As a first remark, one can state that the model's fit for Sweden is much better than for Norway since the explained deviance amounts to 69.7%. The smooth functions of inflation and real output growth forecasts show evidence in favor of a nonlinear policy behavior of the Central Bank, even though the output growth responsiveness is not significant as has been found for Norway. The estimated degrees of freedom of the inflation and output growth smooth functions are 2.53 and 1.74 respectively. An important feature of the policy strategy in Sweden is that the Riksbank's inflation responsiveness is increasing over the entire support of the inflation projections. However, for any forecasts that are above 7% the speed of increase in the inflation reaction seems to be curbed out. As regards the reaction to the output growth expectations, it is increasing for any forecasts up to 1% and then is slightly declining for any projections which are above that level. Therefore, the Riksbank features some asymmetric policy responsiveness to the output growth forecasts around a threshold level of close to 1%. Besides, this asymmetric behavior has to be interpreted with caution since the estimated smooth function is not statistically significant.

Furthermore, the Central Bank has sufficiently raised the policy rate in order to increase the real rate in the event of a rise in the inflation forecasts for any levels that are located in

the -1% to 3% interval. Hence, the Taylor Principle has been satisfied within this forecast range. Nevertheless, for higher inflation projections, the Central Bank has raised the policy rate less than proportionally implying a destabilizing effect on inflation expectations. This evidence suggests that over part of the sample the Riksbank has not managed to properly anchor the inflation expectations with its inflation objective, which could have potentially threatened its overriding price stability goal. However, once inflation has been brought down to levels consistent with its mandate of price stability the Riksbank has exerted a significant and stabilizing effect on inflation expectations.

Finally, consistently with what one would expect for neighboring countries, notice that the policy behavior of the Bank of Norway and the Riksbank are very similar, particularly as regards their inflation responsiveness functions which feature the same estimated degrees of freedom. Nevertheless, the two policies differ in that the Riksbank has adopted a more hawkish policy stance with respect to inflation compared to the reaction of the Bank of Norway.

4.2.7 Switzerland

The Swiss National Bank (SNB) conducts monetary policy in the general interests of the country. In that perspective, the monetary policy strategy embeds an explicit definition of price stability, a medium term inflation forecast and a target range for the three month Libor rate on the Swiss franc as the key policy interest rate. In line with its price stability goal the SNB should also take into consideration the economic outlook as outlined in the Central Bank Act.

The definition of price stability corresponds to an annual increase of CPI inflation of less than 2% that the SNB seeks to achieve over the medium and long terms. Even though the monetary policy framework closely resembles that of an inflation targeting Central Bank, the SNB does not implement an explicit inflation targeting strategy. Nevertheless, the Central Bank considers the inflation forecast as the main indicator for its policy rate decisions, as well as an important communication tool with the relevant market participants.

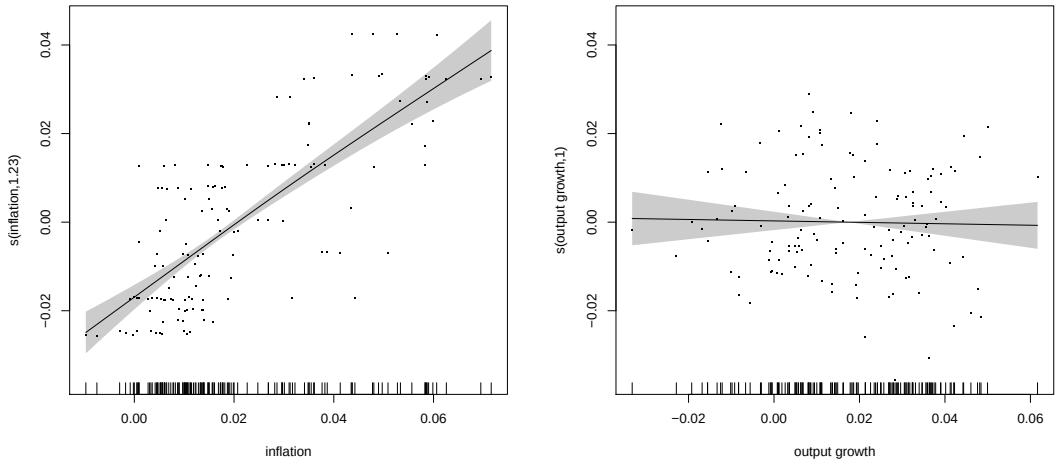


Figure 7: Estimated smooth functions for Switzerland

Switzerland is the country which features a nearly linear Taylor Rule. Indeed, both the inflation and output growth estimated smooth functions, as shown in figure 7, bring evidence in this direction. The estimated degrees of freedom amount to 1.23 and 1 respec-

tively for the inflation and output growth forecasts. Furthermore, the SNB might not have responded to the economic outlook as the estimated smooth function is nearly flat and not statistically significant. The model fits quite well the data featuring an explained deviance of 58.5%.

Given that the Swiss monetary policy rule is the closest to a linear one, it is particularly valuable to estimate the Taylor Rule with the method of Ordinary Least Squares (OLS) in order to explicit the economic intuition about the semi-parametric technique used in the paper. Figures 37 and 38 in the appendix display the partial regression plots for Switzerland performed with OLS. A comparison of the latter with figure 7 yields compelling results. Indeed, as one would expect in this particular case the empirical findings are very similar whether using the GAM's methodology or the simpler parametric OLS regression approach. As the Swiss Taylor Rule is close to a linear policy reaction function, one would expect to estimate a similar policy responsiveness of the SNB to the inflation and output growth forecasts to the one obtained with the GAM's when performing an OLS regression. Furthermore, the confidence intervals in both procedures are of similar magnitude. The tighter confidence intervals obtained in the GAM result from an identifiability constraint which is imposed on the smooth terms.[17] Hence, this evidence highlights the fact that the semi-parametric regression approach adopted in the paper is a relevant technique for estimating the policy reaction functions and produces accurate results which are in line with the intuition one can get from a parametric modeling framework.

It is important to emphasize that the Swiss Taylor Rule is the closest to a standard linear monetary policy rule among the countries studied in this paper. Indeed, the estimated smooth inflation function indicates that the SNB features an increasing responsiveness to the inflation forecasts that has not really changed along the level of the fundamentals. Overall, one can state that the monetary policy strategy has been particularly successful during the entire period investigated since the country has not experienced any high inflationary expectations, even during the 1980's and contrarily to the inflation experience of most of the other industrialized economies. The quite low and stable inflation expectations have remained well anchored to the SNB's definition of price stability and reflect the low inflation credibility the monetary authorities benefit from the market participants.

A careful inspection of the previous figure reveals that surprisingly the SNB has not reacted as strongly to inflation expectations as one would expect. Indeed, the Central Bank responds almost proportionally to changes in the inflation expectations especially for low forecast levels that are between -1% and 2% and reacts clearly less than proportionally to higher inflation projections. Therefore, the evidence on the Taylor Principle is quite mixed in Switzerland suggesting that the latter might not have been satisfied over some of the support of the inflation forecasts. As regards the responsiveness to the economic outlook, the latter might have been slightly destabilizing, but more importantly it shows little evidence about the relevance of the output growth forecasts in the monetary policy strategy.

4.2.8 USA

The U.S. Federal Reserve (Fed) has been assigned with a dual mandate of price stability and maximum employment which is embedded in the Humphrey-Hawkins Act of 1977 and its subsequent amendments: the Fed *"...shall maintain long run growth of the monetary and credit aggregates commensurate with the economy's long run potential to increase production, so as to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates"*. In addition, as in Japan, the Federal Reserve Act does not specify any numerical value for the definition of price stability, leaving this issue to the

[17] This constraint only applies when the estimated degrees of freedom are one. In this case, there exists a point on the empirical support in which the confidence interval is zero.

appropriate judgment of the FOMC's members. Without an explicit inflation targeting framework, the Fed sets a target for the overnight federal funds rate in order to achieve the policy objectives it has been assigned with by the Central Bank Act.

The evidence on the nonlinearity of the Taylor Rule in the USA, as shown in figure 8, is quite mixed. Indeed, on the one hand the estimated smooth function of the inflation forecasts points to a linear relationship with an estimated degree of freedom of one. On the other hand, the estimated smooth function of the output growth forecasts reveals that the relationship is more likely to be nonlinear. The estimated functions are both statistically significant and the explained model deviance is 61.5% which accounts for a good model's fit.

The figure below displays an increasing linear responsiveness of the Fed to the inflation forecasts over the entire range of the variable considered in the estimation which is in line with Taylor (1993). As regards the Taylor Principle, a detailed analysis of the graph indicates that the Fed has adjusted the policy rate almost proportionally to changes in inflation expectations thus bringing a mixed evidence on whether it has exerted a stabilizing effect on inflation expectations and ultimately on the actual rate of inflation.

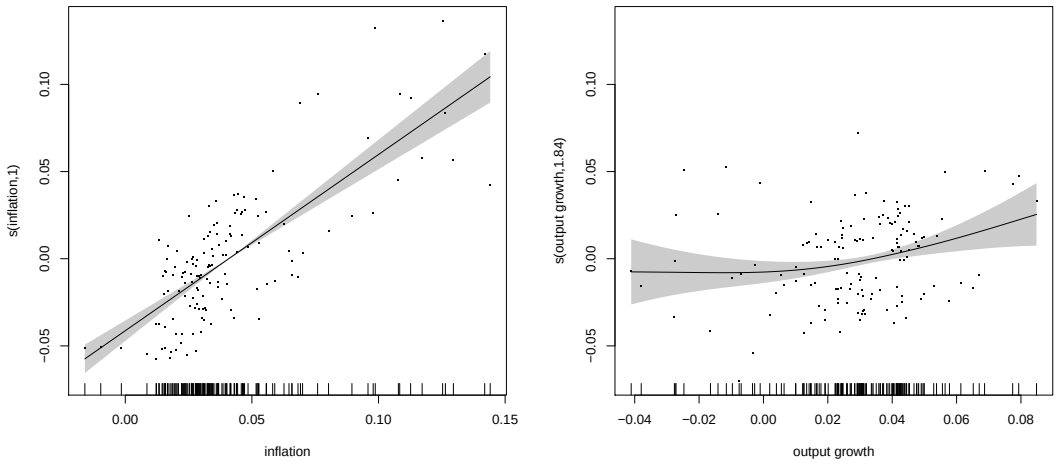


Figure 8: Estimated smooth functions for USA

Regarding the responsiveness to the output growth projections, the latter has changed along the level of the forecasts. It has been particularly flat and not significant for any forecasts between -4% and close to 2%. Then, for any growth projections above the 2% level the responsiveness gradually increases pointing to a stabilizing policy for the economic outlook. The threshold level of 2% is not very surprising as it has often been found to correspond with the growth level of potential output in the US. Hence, if real output growth is below the potential output growth rate the Fed doesn't implement a stabilizing policy as it is concerned about the economic outlook. However, as long as the former exceeds the latter, the Fed adopts a more hawkish policy stance by raising the policy rate. This behavior helps to secure the price stability commitment of the Central Bank, consistently with its dual policy mandate.

Lastly, the predicted policy rate indicates that with deflationary expectations and an output growth forecast close or slightly below the level of 2% the U.S. monetary policy might be facing the zero lower bound problem as it has recently experienced.

To sum up, we would like to emphasize that the univariate GAM presented in this section features some shortcomings compared to an optimal policy rule that one could derive from the theoretical framework of the previous section. First, the model previ-

ously presented does not account for any potential interaction between the inflation and output growth forecasts (non-zero cross-derivatives). This reflects the trade-off monetary authorities face in the event of a cost-push shock for instance and that might imply a nonlinear policy response. Second, the above univariate GAM cannot explain a possible change of regime due to the adoption of a new policy concept or related to a change in the Chairman of the monetary policy committee for instance. Therefore, in the forthcoming subsection we augment the baseline univariate specification with an interaction term between the inflation and output growth forecasts, as well as with interactions between the forecasts and time. The latter are aimed at accounting for a possible change of regime in monetary policy. Indeed, with the augmented specification we can bring to light the fact that the policy response of the Central Bank could differ for the same level of economic fundamentals depending on the specific time period that one is looking at. Thereby, with this augmented specification we corroborate the evidence for the nonlinearity of the Taylor Rule for all the countries including Switzerland as well.

4.3 Univariate GAM with interaction terms

The optimal policy rule derived from the theoretical model shows the relevance of an interaction term between the inflation and the output gap forecasts. This theoretical underpinning is further corroborated from the perspective plots of the fitted policy rates with the bivariate GAM that have been presented in the first subsection. Therefore, this part of the paper is devoted to investigating more in-depth the specification of the Taylor Rule by introducing some relevant interaction terms in the regressions. We thus augment the baseline specification to the following one:

$$i_t = c + s_1(E_t\pi_{t+1}) + s_2(E_t x_{t+1}) + s_3(E_t\pi_{t+1} \times E_t x_{t+1}) + s_4(E_t\pi_{t+1} \times t) + s_5(E_t x_{t+1} \times t) + \epsilon_t \quad (16)$$

where $s_1(E_t\pi_{t+1})$ and $s_2(E_t x_{t+1})$ denote the smooth functions of the inflation and output growth forecasts respectively. $s_3(E_t\pi_{t+1} \times E_t x_{t+1})$ refers to a smooth function of the interaction between the inflation and output growth forecasts, while $s_4(E_t\pi_{t+1} \times t)$ and $s_5(E_t x_{t+1} \times t)$ denote the smooth functions of the interaction between the inflation forecasts and time, and the output growth forecasts and time respectively.

The first interaction term is derived directly from the theoretical framework and the evidence on the bivariate GAM. It accounts for the effect of persistent shocks on the Central Bank's policy stance. Indeed, in the event of a cost-push shock the Central Bank faces a trade-off between inflation and output growth stabilization. On the one hand, the inflation forecast increases, while on the other the output growth forecast evolves in the opposite direction. Therefore, the interaction term captures the dilemma the Central Bank faces about either raising the policy rate to prevent the build-up of inflationary pressures or pushing down the interest rate to stabilize the economic outlook. The interactions between the inflation and output growth forecasts with time are modeled to account for the corresponding business cycle and are justified from a time series modeling approach. Indeed, the Central Bank will be more willing to increase the policy rate in a monetary policy tightening cycle than in a cycle of policy easing, everything else being equal. The closest analog is the interest rate smoothing practice of Central Banks that has been accounted for in the empirical literature of Taylor Rules. However, we agree with Rudebusch (2002 and 2006) who criticizes the use of interest rate smoothing in monetary policy rules. The latter states that the high predictive ability of Taylor Rules with interest rate smoothing stands in sharp contrast with the evidence from the term structure of interest rates. Hence, according to Rudebusch the empirical evidence for interest rate smoothing might result from a misspecification of the Taylor Rule and reflects the persistent shocks that Central Banks face. In his paper of 2006 he concludes by asserting the following: *"...In essence, quarterly*

monetary policy partial adjustment does not appear to be consistent with the financial market's understanding of the monetary policy rule. This absence of intrinsic inertia appears in accord with the views of many central bankers, who often note that future policy actions will largely be contingent on incoming data and future changes in the economic outlook." Besides, by explicitly introducing the time dimension in the estimations we can formally account for a structural break embedded in the series stemming either from a change in the Chairman of the policy committee or from a change in the monetary policy regime for instance.[18]

Specification (16) is also the favorite one on the statistical ground. Table 7 in the appendix reports the output of the estimations.[19] Indeed, it features the lowest Akaike values compared to specification (15) for all the Central Banks included in the analysis. Moreover, the explained deviance of the model sharply increases with specification (16) and the estimated smooth functions of the interaction terms are highly statistically significant for all the Central Banks' reaction functions, except for one interaction term in Norway. The most striking increase in the model's fit is observed for Norway and Sweden. Indeed, the table points out that in the former country the explained deviance more than doubles since it increases from 25.5% to 52.2%, while in the latter it increases from 69.7% to 93.8%.

Furthermore, in order to account for the model accuracy in predicting the actual pattern of the policy rates, figures 39(a) to 39(h) in the appendix, display the graphs of the fitted policy rates with the above GAM specification together with the actual policy rate set by the corresponding Central Bank. In order to provide a benchmark comparison each graph also displays the fitted values from a standard linear Taylor Rule specification, as well as the fitted values from a GMM regression that includes interest rate smoothing in the policy reaction function. The latter model takes the following form:

$$i_t = \rho i_{t-1} + (1-\rho)[\alpha + \beta_\pi E_t \{\pi_{t+1}|\Omega_t\} + \beta_x E_t \{x_{t+1}|\Omega_t\}] + \epsilon_t \tag{17}$$

where ρ denotes the degree of policy inertia, α is a constant term and ϵ_t is a stochastic disturbance. In the GMM estimations, three lags of the exogenous variables have been used as instruments for the lagged dependent variable. In the regressions, the Hansen's J-statistic does not show evidence against the validity of the instrumental variables for all countries.

As a first remark, it is compelling to notice that the above GAM specification performs remarkably well across all the countries considered in the regressions. Second, a robust result is that for all countries the augmented GAM specification performs better than the standard linear Taylor Rule. Indeed, the fitted values obtained with the former are much closer to the actual policy rate than the fitted values from the latter. However, Switzerland is an exception since the performance of the GAM's Taylor Rule is quite close to the one with the linear specification. Third, as regards the comparison of the GAM's fitted values with the predictions from the model with interest rate smoothing, the performance of the augmented GAM is very close to the one of the GMM model for most of the countries.

Regarding monetary policy in Australia, the GAM fits much better the actual policy rate than the standard linear Taylor Rule over the last three decades. In particular, notice that at the end of the 1970's the linear reaction function has recommended a policy rate that is too high relative to the predictions of both the GAM and the specification with interest rate smoothing. Similarly, the linear Taylor Rule would have recommended a policy rate that is too low relative to the other models' predictions for the period 1985-1990. Furthermore, the GAM performs quite well in comparison to the model with interest rate smoothing for some periods, in particular at the end of the 1970's, at the end of the

[18] For the same values of economic fundamentals, specification (16) allows us to account for a possibly different policy responsiveness for different time periods.

[19] Further details on the regressions are available from the authors on demand.

1980's and at the beginning of the last decade. Finally, in light of the GAM predictions the current policy stance in Australia seems to be appropriate.

The estimated policy rate from the GAM for Canada clearly outperforms the predictions from the linear specification. In fact, at the end of the 1970's the linear Taylor Rule predicts a policy stance that is too tight compared to the predictions from the other two models. In the same spirit, the linear Taylor Rule points out that the policy rate should have been lower during most of the 1980's and higher at the beginning of the last decade, a result which stands in contrast with the other models' predictions. An important finding is that the GAM specification performs well in terms of the interest rate predictions even when compared to the predicted interest rates from the model with interest rate smoothing. Finally, notice that the GAM points out that the current policy rate in Canada should be close to 1.25%, which is a level just above the policy rate of 1% in December 2010. Note that this result is close to the predictions from the model with policy inertia which considers the current level of the policy rate as appropriate but stands in contrast with the predictions from the linear policy rule.

The predictions of the GAM specification are particularly well aligned with the actual policy rate pattern in Japan. Indeed, the semi-parametric specification performs much better in terms of the model's fit than the linear Taylor Rule over the entire period. Moreover, the fitted policy rate from the GAM is close to the predictions from the model with interest rate smoothing. The standard Taylor Rule predicts that the policy rate should have been higher at the end of the 1970's and monetary policy should have been more accommodative during the 1980's. In addition, the Bank of Japan should have implemented a tighter policy stance during most of the 1990's and over the last decade. An interesting finding is that the semi-parametric policy rule points out that the current policy stance seems to be appropriate in light of the economic fundamentals. This result is partly corroborated from the predictions of the specification with policy inertia and stands in contrast with the evidence from the linear Taylor Rule model.

The performance of the semi-parametric Taylor Rule for New Zealand is particularly compelling compared to both the linear Taylor Rule and the specification with policy inertia. A striking result is that for most of the period considered in the analysis the GAM specification performs even better in terms of the interest rate predictions than the model with interest rate smoothing. Hence, monetary policy in New Zealand could be more accurately described by a semi-parametric policy reaction function that contains relevant interaction terms than by a linear parametric Taylor Rule with policy inertia. Notice that the current policy stance seems to be too accommodating compared to the interest rate predictions from the GAM specification, the linear Taylor Rule and the specification with interest rate smoothing. Therefore, given that the three models' predictions point in the same direction, the RBNZ should have increased the policy rate in 2010 to prevent the build-up of inflationary expectations in the economy.

Norway is the country in which the linear Taylor Rule features the worst fit with the actual policy rate. Indeed, the semi-parametric Taylor Rule performs clearly better than a linear rule and the recommended policy rate is the closest to the one from a parametric rule with policy inertia except for the periods that feature large swings in the interest rate. The fitted rate from the GAM specification closely follows the pattern of the actual policy rate, while the linear Taylor Rule implies that the Bank of Norway should have implemented a much smoother policy stance since the second half of the 1980's compared to the one it has been following. In particular, as regards the current policy stance, both the linear and semi-parametric policy rules point out that it should be tighter. The Taylor Rule with policy inertia also suggests that the Central Bank should have raised the policy rate in 2010 but to a much lesser extent.

Concerning the Swedish Riksbank, the linear Taylor Rule model displays the worst

fit among the three specifications considered. The semi-parametric Taylor Rule closely follows the observed pattern of the actual policy rate and performs quite well even when compared to the predictions from the Taylor Rule with interest rate smoothing. As for the Central Banks previously analyzed, the linear Taylor Rule points out that monetary policy might have been too accommodating by the end of the 1970's and too tight by the end of the 1980's. The current policy stance of the Riksbank seems to be quite appropriate even though the nonlinear Taylor Rule points out that the policy rate should be around 0.5%, which is below its level of 1% by the end of 2010. The rule with policy inertia shows that the current policy stance is fully appropriate, while this evidence stands in sharp contrast with the predictions from the linear Taylor Rule. In fact, the latter suggests that monetary policy should be much tighter than the actual policy stance in 2010.

Switzerland is the country in which the predictions from the semi-parametric Taylor Rule are the closest to those from a linear reaction function. As illustrated by the graph in the appendix, shown in figure 39(g), the best fit is obtained from a linear Taylor Rule with interest rate smoothing. However, there are important differences in the models' predictions for some periods. Indeed, both the linear and semi-parametric reaction functions point out that monetary policy might have been too accommodating at the end of the 1970's and too tight during the 1980's in contrast to the predictions from the Taylor Rule with interest rate smoothing. Furthermore, the fitted values from the linear and semi-parametric models indicate that at the beginning of the last decade the policy stance might have been particularly accommodating. Finally, the current level of the policy rate could be too low compared to the predictions from both the linear and the GAM specifications. In fact, the predicted policy rates suggest that the target for the 3 month Libor rate on the Swiss franc should be close to 75 basis points at the end of 2010 which corresponds to the upper bound of the target range of the SNB. This result contrasts with the evidence from the model with policy inertia.

The semi-parametric Taylor Rule for the Federal Reserve matches overall well the observed behavior of the fed funds. In particular, it provides a better fit to the actual policy rate compared to the predictions from the linear model. The linear Taylor Rule points out that the U.S. monetary policy might have been too accommodating at the end of the 1970's and too tight during the 1980's, a result that has been found for all the above Central Banks. Another interesting finding is that the fed funds have remained at a too low level for an extended period of time at the beginning of the last decade compared to the predictions stemming from the linear and semi-parametric Taylor Rules. Finally, the current policy stance seems to be overly expansionary in the United States as well. Indeed, both the linear and semi-parametric policy rules point out that the appropriate level of the fed funds should be clearly above 1% in 2010, which is in stark contrast with the actual policy rate that lies at the vicinity of the zero lower bound. Hence, if the Fed maintains such an expansionary policy for a prolonged period of time, it could pave the way for increasing inflationary pressures and favor the emergence of potential asset price misalignments in the economy.

5 Out-of-sample forecasting

In this part of the paper we test the forecasting ability of the augmented nonlinear Taylor Rule. This exercise will shed more light on determining the model's accuracy and stability in forecasting the future interest rate path of the OECD Central Banks out of the sample used for the estimations. This approach is justified both from a statistical and a practical policy making perspectives. Indeed, on the one hand the former consists in a cross validation method that performs model selection. On the other hand, a good model performance in out-of-sample forecasting is a valuable feature that permits a practical application of

the model by Central Bankers for taking appropriate decisions on the policy interest rate using the most accurate information at their disposal.

We have performed two types of model forecasting: a forecast of the policy interest rate for the period 2010 Q1 to 2010 Q4 using all the in-sample information set until 2009 Q4, as well as a one-quarter ahead rolling window forecast starting in 1996 Q3 and ending in 2010 Q4. The former specification takes the following form:

$$\begin{aligned}\hat{i}_{T+n|T} &= \hat{c} + \hat{s}_1(E_T\pi_{T+n}) + \hat{s}_2(E_T x_{T+n}) + \hat{s}_3(E_T\pi_{T+n} \times E_T x_{T+n}) \\ &\quad + \hat{s}_4\big(E_T\pi_{T+n} \times (T+n)\big) + \hat{s}_5\big(E_T x_{T+n} \times (T+n)\big) \end{aligned} \tag{18}$$

for $n = 1, ..., 4$ and T=134 (2009 Q4).

In a similar way the rolling window projection of the policy rate for the period 1996 Q3 to 2010 Q4 can be written as follows:

$$\begin{aligned}\hat{i}_{T-n+1|T-n} &= \hat{c} + \hat{s}_1(E_{T-n}\pi_{T-n+1}) + \hat{s}_2(E_{T-n}x_{T-n+1}) \\ &\quad + \hat{s}_3(E_{T-n}\pi_{T-n+1} \times E_{T-n}x_{T-n+1}) + \hat{s}_4\big(E_{T-n}\pi_{T-n+1} \times (T-n+1)\big) \\ &\quad + \hat{s}_5\big(E_{T-n}x_{T-n+1} \times (T-n+1)\big) \end{aligned} \tag{19}$$

for $n = 1, ..., 58$ and T=138 (2010 Q4).

Since the inflation and output growth forecasts are not known out-of-sample for the periods we forecast we have modeled an explicit stochastic process for these variables. Indeed, consistently with most of the empirical literature and with our in-sample data, we have specified a random walk process with heteroskedastic error terms for both the inflation and output growth forecasts.[20] This is a reasonable scenario given that in practice it is quite difficult to ascertain the way expectations of macroeconomic variables are generated. Formally, the latter can be expressed as:

$$E_t\pi_{t+1} = E_{t-1}\pi_t + \epsilon_t^{\pi}, \epsilon_t^{\pi} \sim N(0, \sigma_{\pi,t}^2) \tag{20}$$

$$E_t x_{t+1} = E_{t-1}x_t + \epsilon_t^{x}, \epsilon_t^{x} \sim N(0, \sigma_{x,t}^2) \tag{21}$$

for the inflation and output growth forecasts respectively.

To insure a proper convergence of the process we have simulated 1000 times the stochastic process of the economic fundamentals using the *R* software. Then, the point forecast of each policy rate has been obtained by computing the average of the simulated forecasts once a trimming of 10% has been applied on the generated series. In order to construct the forecast intervals we have assumed normality of the predicted policy rates. Finally, in line with the forefront research in terms of policy communication and transparency we have reported the fan charts for each point forecast in order to describe the uncertainty surrounding the projections. The reported confidence intervals range from 30% to 90% and mirror the current practice of the Bank of Norway and the Swedish Riksbank regarding their communication of the expected interest rate path. The generated forecasts, denoted by TR GAM, are displayed in figures 40(a) to 47(d) in the appendix. In order to provide some benchmark comparison of the forecast accuracy of the augmented GAM we have also performed the forecasts using the bivariate GAM, the univariate GAM without interaction terms as well as the standard linear Taylor Rule specification. The Root Forecast Mean

[20] In view of generating the process we estimate the variance of the two random walks over a period of ten and five years prior to the time periods T and $T-n$ in specifications (18) and (19) respectively.

Squared Error (RFMSE) of each model is reported in table 8 in the appendix. The latter provides useful information for the out-of-sample model selection. The table points out that there is a substantial gain in producing out-of-sample forecasts of the policy rate with a semi-parametric rather than with a linear model. In fact, depending on the forecasting procedure used, for some countries the semi-parametric approach yields some forecasts that are several times more accurate than the projections obtained with a linear policy rule. The most striking example is Japan where the forecast of the policy rate for 2010 obtained with the GAM is almost 264 times more accurate than the one produced with the linear model, followed by Australia and Norway to a lesser extent. This evidence highlights the importance of using the semi-parametric models to produce more accurate out-of-sample forecasts of the policy rates for all the countries considered in the sample.

The augmented GAM performs particularly well for Australia using both forecasting procedures. Indeed, as one can see from figures 40(a) and 40(b) the forecasted policy rate for 2010 is very well aligned with the actual policy rate suggesting that we could have quite accurately predicted the policy stance in Australia using the out-of-sample forecasts of inflation and real output growth. This highly predictive ability of our model stands in contrast with the projections from the linear Taylor Rule. Indeed, the latter point out that the policy rate should have been much tighter during the entire forecasting period. In addition, figures 40(c) and 40(d) corroborate these findings for the rolling window forecasting procedure. Hence, almost the entire pattern of the actual policy rate lies within the forecast interval of the GAM Taylor Rule contrarily to the evidence from the linear model. Finally, the comparison of the RFMSE reveals that the augmented GAM features the best forecasting performance among all four models.

Figures 41(a) and 41(b) display the forecasted policy rate in Canada for 2010. The nonlinear specification features a good forecasting performance compared to the linear model, especially during the first two quarters of 2010. In terms of the RFMSE the best forecasting model is the bivariate GAM, while the linear model performs slightly worse than the augmented GAM. As regards the rolling window forecasts which are reported in figures 41(c) and 41(d), the GAM Taylor Rule follows more closely the actual policy rate path than the linear Taylor Rule for the entire period. Indeed, the augmented GAM features the best performance in terms of the lowest RFMSE compared to the performance of the other models.

A closer look at figures 42(a) and 42(b) in the appendix shows that the GAM accurately predicts the very low level of the actual policy rate for 2010 in Japan. Indeed, the semi-parametric model forecasts better and with less uncertainty the policy rate compared to the predictions from the linear model. The latter even points to highly negative nominal interest rates in Japan. In terms of the RFMSE the GAM performs better than the linear model but the best forecasting model is the univariate GAM without interaction terms. Regarding the rolling window predictions shown in figures 42(c) and 42(d), the GAM point forecast is close to the actual policy path except during two major crises: the first one corresponds to the Asian financial crisis of 1997-98, while the second refers to the recent subprime turmoil. The interest rate forecasts are associated with a high level of uncertainty during both crises which stands in contrast with the predictions from the linear model. In terms of the RFMSE performance the best model is a bivariate GAM.

Figures 43(a) and 43(b) point out that it is quite challenging to predict the policy rate in New Zealand. Even though the point forecast of the GAM gets closer to the actual interest rate over time it predicts a tighter policy stance compared to the actual one. The gap between the actual and forecasted interest rates is even sharper when using the predictions from the linear model. The graphs thus show that the actual policy stance in New Zealand is particularly accommodative compared to the predictions one could have made using the generated forecasts of inflation and output growth. As regards the RFMSE the augmented

GAM scores the best. Within the rolling window methodology the GAM forecasts feature a high level of variability and uncertainty compared to the linear model. In fact, as one can see from figures 43(c) and 43(d) there are some important swings in the point forecasts particularly during the 2000-2001 crisis and during the recent financial turmoil compared to the linear model predictions. Besides, the latter feature some important gaps between the point forecasts and the policy rate. In terms of the RFMSE criterion the univariate GAM without interactions performs the best.

The GAM forecasts of the policy rate in Norway reveal that the interest rate should be set at the vicinity of the zero lower bound contrarily to the predictions from the linear model. The projections are displayed in figures 44(a) and 44(b). However, while the latter point forecasts are particularly accurate, the former are surrounded by a high level of uncertainty. In terms of the RFMSE benchmark the best performance is attributed to the bivariate GAM, while the linear model features the worst fit. Concerning the forecasts from the rolling window approach, presented in figures 44(c) and 44(d) an important finding is that the GAM has predicted the sharp fall of the policy rate at the tipping point of the recent crisis by the end of 2008. This evidence stands in contrast with the projections from the linear model which point to a tighter policy stance during most of the recent decade. The model scoring the lowest RFMSE is the GAM with interactions among the four models considered in the forecasting procedure.

The predictions of the augmented GAM displayed in figure 45(a) exhibit a high level of uncertainty in Sweden. Besides, the latter point out that the policy rate should be set at the zero lower bound during most of the year. This evidence contrasts with the point forecasts from the linear model shown in figure 45(b). The latter reveal that using the out-of-sample forecasts of inflation and real output growth the Riksbank should have implemented a tighter policy stance compared to the actual one in 2010. In terms of the RFMSE criterion the best performing model is the univariate GAM without interactions. Figures 45(c) and 45(d) present the rolling window forecasts of the policy rate. It appears that the augmented GAM performs the best in terms of the RFMSE criterion. Indeed, it predicts an interest rate which is the closest to the actual policy rate path when compared to the forecasts from the linear model. The former has also accurately forecasted the sharp fall of the interest rate at the peak of the recent crisis.

The predictions of the policy rate in Switzerland for 2010 are displayed in figures 46(a) and 46(b). The latter show that within the out-of-sample framework one should have implemented a tighter policy stance in 2010 compared to the actual policy rate either using the GAM or the linear model. The best performance in terms of the lowest RFMSE is attributed to the bivariate GAM. In addition, figures 46(c) and 46(d) display the rolling window forecasts of the policy rate. In general, the latter show that the actual interest rate path lies more within the GAM forecasts than within the linear model projections. Moreover, both model forecasts indicate that the policy stance should have been tighter from 2002 to 2006 compared to the actual one. Finally, the GAM has predicted well the sharp decrease of the policy rate in the first quarter of 2009, while the linear model has forecasted the rate cut with a substantial delay. In terms of the RFMSE the best forecasting model is the bivariate GAM.

The forecasts of the federal funds rate for 2010 are shown in figures 47(a) and 47(b). The out-of-sample GAM predictions of the fed funds indicate that the policy rate should have been set at the zero lower bound consistently with the actual interest rate pattern. This prediction contrasts with the forecasts from the linear model which point out that the actual policy stance is too accommodating and thus the fed funds should have been raised at a level close to 3% in 2010. Besides, the best forecasting performance in 2010 is attributed to the augmented GAM specification. Figures 47(c) and 47(d) display the rolling window forecasts of the fed funds. The latter show that the policy rate lies much more in

the GAM predictions than within the linear model projections. In addition, both models reveal that the actual fed funds should have been higher from 2001 to 2005 compared to their observed pattern. The GAM Taylor Rule points out that the policy rate should have been higher in 2010 which is in line with the linear model forecasts. Finally, the augmented GAM scores the best in terms of the RFMSE criterion for the rolling window forecasts as well.

To sum up, the previous evidence suggests that the augmented GAM performs well in forecasting out-of-sample the actual interest rate pattern of the OECD Central Banks. Even though in some countries the best model in terms of the lowest RFMSE criterion is found to be either the bivariate GAM or the univariate GAM without interactions, one can clearly reject the linear Taylor Rule model as it performs the worst in predicting the policy interest rate among all the countries investigated and within both forecasting procedures. Importantly, for most of the countries, the out-of-sample forecasts are in line with the in-sample predictions which accounts for a good model's accuracy.

6 Robustness analysis

As a sensitivity analysis, all the regressions have been performed with the Hodrick-Prescott (H-P) detrended change in the output gap in the spirit of Walsh (2003).[21] The speed limit policy he proposes introduces some inertia that improves the trade-off between inflation and output gap variability and is preferable to a policy that considers a standard measure of the output gap. The estimation results are reported in tables 9 to 11 in the appendix. In line with the results from the baseline model with output growth, we find a clear and significant evidence for the nonlinearity of the Taylor Rule for all the countries investigated. Consistently with the previous findings, table 9 shows that the country with the lowest degree of nonlinearity is Switzerland with an estimated degree of freedom of $s(E_t\pi_{t+1}, E_t x_{t+1}) = 2.49$. However, the highest degree of nonlinearity is reported for the USA which feature an estimated degree of freedom of $s(E_t\pi_{t+1}, E_t x_{t+1}) = 18.59$. A comparison of figures 9 to 16 along with figures 17 to 24 shows that the responsiveness of the policy rate to the forecasts of inflation and to the change in the output gap projections are similar for Australia, Canada, Japan and Switzerland. The shape of the estimated figures is different for New Zealand, Norway, Sweden and the USA mostly with respect to the forecasts of the economic outlook. Besides, the results suggest a possible interaction between the inflation and output gap change forecasts for all countries except Switzerland, which is consistent with the earlier findings.

The regressions of the GAM without interaction terms are reported in table 10. Overall, the estimation results are qualitatively unaltered for Australia, Japan, New Zealand, Norway, Sweden and Switzerland. However, for Canada the responsiveness to the output gap change forecasts is linear while for the USA the estimated smooth term of the inflation forecasts is nonlinear which differs from the baseline results. The explained deviance of the model is smaller for most of the countries and from an information criterion perspective one should prefer the specification with the output growth forecasts except for Norway and Sweden.

The results from the GAM specification with interaction terms are displayed in table 11. The latter points to the relevance of the interaction terms which are highly significant for most of the countries. In New Zealand and Switzerland only the interaction term of the inflation forecasts with time is significant. The model's in-sample predictions performed

[21] The H-P filter is applied to the real output growth forecasts in order to derive the growth rate of potential output. A smoothing parameter of 1600 has been used to account for the quarterly frequency of the data. Then, the change in the output gap is obtained by subtracting the growth rate of potential output from the real output growth rate.

with the change in the output gap are broadly aligned with the predictions obtained from the baseline model for most of the countries. In particular, the in-sample fit confirms that the policy stance is broadly appropriate in Australia, Canada, Japan and Sweden while it should be tighter in New Zealand, Norway, Switzerland and the USA in 2010 as previously recommended. According to the Akaike's criterion, the baseline model with the output growth forecasts should be preferred to the model with the H-P detrended change in the output gap forecasts for all countries in our data set.

Finally, the out-of-sample root forecast mean squared errors of the semi-parametric and linear models, which are reported in table 12, corroborate the predictions from the models with the output growth forecasts. In particular, the nonlinear specification systematically outperforms the linear Taylor Rule for all countries within both forecasting procedures. Besides, the table shows more evidence in favor of the augmented GAM specification as it scores the best in forecasting the policy rate in all countries except in Switzerland for the 2010 forecast horizon. As regards the rolling window horizon the augmented GAM features the best forecasting performance in most of the countries except in New Zealand, Norway and Switzerland. In the latter countries the bivariate semi-parametric model performs the best in forecasting the policy rate. A comparison of tables 8 and 12 shows that for Australia, Switzerland and USA the best forecasting model is the same regardless of using either the output growth or the change in the output gap forecasts in the regressions. In Canada and Sweden the augmented GAM scores the best in the rolling window forecasts, while for New Zealand it features the best performance for the 2010 forecast horizon in both the baseline and in the robustness regressions.

7 Estimations with the consensus forecasts

In this section we present the empirical evidence for the previous specifications using Consensus Economics Forecasts of inflation and real GDP growth for Canada, Japan, Norway, Sweden, Switzerland and USA.[22] The goal is to determine whether the semi-parametric models perform well in estimating and in forecasting out-of-sample the Taylor Rules for the previous countries using an alternative database. The interest in this analysis lies in the data which are provided in real-time and thus could have been used by Central Bankers for estimating a general nonlinear specification of the Taylor Rule and forecasting the future policy interest rate path. The estimation results are reported in tables 13 to 20 in the appendix.

The perspective plots of the estimated smooth functions of the bivariate GAM are reported in figures 25 to 30 in the appendix. The latter show clear evidence for a nonlinear Taylor rule and point to the presence of possible interaction between the inflation and real output growth forecasts for all countries. The degree of nonlinearity of the policy rule is quite high for all countries ranging from 17.56 to 25.70 estimated degrees of freedom of the smooth functions for Canada and Switzerland respectively. The model performs well since the explained deviance is high for all countries. The latter ranges from 74.9% for Norway to 96% for Switzerland. In contrast with the previous findings, Switzerland appears as the country with the highest degree of nonlinearity when using the consensus forecasts in the estimations. Notice also that the model captures well the zero lower bound problem the Japanese authorities have been facing in the second half of the 1990's and during the last decade as shown in figure 26. Finally, figures 31 to 36 in the appendix corroborate the evidence for the nonlinearity of the Taylor Rule for all countries using the change in the output gap instead of the output growth rate in the regressions. In Switzerland and in the USA the perspective plots suggest that the policy rate is decreasing in the change of

[22] The consensus forecasts for Australia and New Zealand are not available since the start of the surveys in October 1989.

the output gap. However, this evidence does not necessarily imply a destabilizing policy because we have to consider the relevant interaction terms in the Taylor Rules as well.

The GAM with no interactions, which is presented in table 14, shows that the estimated smooth terms are highly significant and point to a nonlinear specification of the policy rules. The results from the augmented GAM are shown in table 15. The latter indicate that most of the estimated interaction terms are highly significant for all countries. Besides, as found with the OECD forecasts, the explained deviance increases substantially when moving from the GAM without interaction terms to the augmented specification for most of the countries. According to the AIC information criterion the GAM with interactions should be preferred for all countries which is consistent with the baseline evidence. Finally, the augmented specification performs better than the linear Taylor Rule in predicting the policy rate within the sample and its performance is close to the one of a policy rule with interest rate smoothing. These results are robust to using the change in the output gap as an alternative regressor as shown in tables 18 and 19.

Finally, the out-of-sample forecast errors are presented in tables 16 and 20. The latter show evidence for a better forecasting performance of the semi-parametric specifications of the Taylor Rule compared to the linear model. Table 16 points out that the GAM specifications perform better in forecasting the policy rate out-of-sample in most of the countries within both forecasting procedures. Notice that there is a very high forecast gain in using a semi-parametric specification for the policy rate in Japan within the four-months ahead forecasting horizon. A comparison of tables 8 and 16 shows that the latter is in line with the high forecast gain previously obtained with the OECD data. There are only two exceptions in Canada and in Switzerland where the linear model performs better in forecasting the policy rate four months ahead, from December 2009 to March 2010. However, the augmented and bivariate GAM score the best in forecasting the policy rate within the rolling window one-month ahead forecasts for all countries. The results from table 20 bring further support for the augmented GAM in forecasting the policy rate out-of-sample. The augmented specification scores the best in all countries within the rolling window one-month ahead forecast horizon except in the USA. In the latter country, the bivariate GAM is the best model for forecasting the policy rate. The linear model performs the best only in Switzerland in predicting the policy rate four months ahead. Overall, the results from the out-of-sample forecasting bring evidence for the semi-parametric specification of the Taylor Rule for most of the countries using the consensus data. In particular, the bivariate and the augmented GAM are the preferred models because they score the highest gain in predicting the policy rate for the majority of the countries investigated.

8 Conclusion

This paper has shed more light on understanding the specification of the Central Bank reaction function. Building on the New Keynesian model we have shown evidence that in a more general framework the Taylor Rule is likely to be nonlinear. The paper has contributed to the literature in the following aspects.

First, we have used a semi-parametric modeling approach to empirically investigate the Taylor Rule specification. The evidence has shown that the Taylor Rule is clearly and significantly nonlinear since the degree of Central Bank's responsiveness changes along the forecasted levels of economic fundamentals. This result is robust for all OECD Central Banks considered in the analysis. Switzerland is the only exception for which the results from the bivariate specification suggest that the policy rate is more likely to be linear using the OECD data.

Second, building on the theoretical framework and on the empirical evidence we have

proposed an augmented specification of the policy reaction function. The latter accounts for the importance of the interaction between the inflation and output growth forecasts, and the interaction between the fundamentals and the business cycle in the prediction of the policy interest rate. The estimation results have pointed out that our specification has followed much closely the actual policy interest rate pattern than the standard Taylor Rule for all the countries investigated. Indeed, the semi-parametric Taylor Rule with interaction terms performs better than the linear Taylor Rule for all countries except for Switzerland where the augmented specification performs closely to the linear reaction function. However, we have shown that the model's fit considerably improves when accounting for relevant interaction terms in the Swiss policy rule. Furthermore, the augmented model is also the best specification on the econometric and statistical grounds. Importantly, the performance of the augmented semi-parametric specification is similar to the predictions derived from a Taylor Rule with interest rate smoothing. Hence, given that the Taylor Rule might suffer from a possible misspecification problem when including interest rate smoothing in the regression, our specification provides a reasonable alternative for determining the appropriate monetary policy stance of Central Banks.

Finally, the out-of-sample forecasts of the policy interest rates suggest that in order to more accurately predict the interest rate pattern of Central Banks, based on the forecasts of economic fundamentals, one should prefer the use of a nonlinear semi-parametric Taylor Rule. Indeed, the GAM specifications systematically outperform the linear Taylor Rule in predicting the policy rate for all countries in our sample. In addition, the robustness analysis has corroborated the evidence in favor of the nonlinearity of the policy rule. It has also confirmed the better forecasting performance of the semi-parametric specifications over the linear Taylor Rule with a particular emphasis on the augmented GAM which scores the best in most of the countries within both forecast horizons. We have also found that for some countries the responsiveness of the policy rate to the economic outlook is particularly sensitive to the measure of trend output growth rate used in the regressions. The estimation results and the out-of-sample forecasts with the consensus data bring further support to the semi-parametric specifications of the policy rule. The latter show evidence that in most of the countries we could have accurately predicted the key policy rate in real-time using either the bivariate, univariate or augmented GAM. Given the strong empirical support for the semi-parametric Taylor Rule, we hope that future research will focus on understanding more in-depth the importance of interaction terms in the general nonlinear modeling of policy reaction functions. Furthermore, following a bottom-up approach it would be valuable to parameterize the nonlinear specification in order to enhance the Central Bank's communication strategy with the relevant market participants. In a future work, it would also be interesting to investigate in more details both theoretically and empirically the sources of nonlinearities we encounter in each country.

References

[1] Alcidi C., Flamini A., Fracasso A. (2005), "Taylored rules. Does one fit (or hide) all?", *HEI Working Paper*, 04/2005.

[2] Assenmacher-Wesche K. (2006), "Estimating Central Banks' Preferences from a Time-Varying Empirical Reaction Function", *European Economic Review*, 50(8), 1951-1974.

[3] Clarida R., Galí J., Gertler M. (1998), "Monetary Policy Rules in Practice: Some International Evidence", *European Economic Review*, 42(6), 1033-1067.

[4] Clarida R., Galí J., Gertler M. (1999), "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature*, 37(4), 1661-1707.

[5] Clarida R., Galí J., Gertler M. (2000), "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory", *Quarterly Journal of Economics*, 115(1), 147-180.

[6] Coibion O. and Gorodnichenko Y. (2011), "Why are Target Interest Rate Changes so Persistent?", *NBER Working Paper*, 16707.

[7] Conrad C., Lamla M., Yu K. (2010), "Non-linear Taylor Rules", *Preliminary Working Paper presented at the CIRET Conference on Economic Tendency Surveys in New York 2010.*

[8] Cukierman A. and Muscatelli A. (2008), "Nonlinear Taylor Rules and Asymmetric Preferences in Central Banking: Evidence from the United Kingdom and the United States", *The B.E. Journal of Macroeconomics*, 8(1).

[9] Dolado, J., Dolores R., Naveira M. (2005), "Are Monetary-Policy Reaction Functions Asymmetric?: The Role of Nonlinearity in the Phillips Curve", *European Economic Review*, 49(2), 485-503.

[10] Galí J. (2008), "Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework", *Princeton University Press.*

[11] Gerlach S. , Lewis J. (2010), "The Zero Lower Bound, ECB Interest Rate Policy and the Financial Crisis", *De Nederlandsche Bank Working Paper*, 254.

[12] Gorter J., Jacobs J., de Haan J. (2008), "Taylor Rules for the ECB using Expectations Data", *Scandinavian Journal of Economics*, 110(3), 473-488.

[13] Hamilton J.D. (2001), "A Parametric Approach to Flexible Nonlinear Inference", *Econometrica*, 69(2), 537-573.

[14] Hastie T. and Tibshirani R. (1986), "Generalized Additive Models (with discussion)", *Statistical Science*, 1(1), 297-318.

[15] Hastie T. and Tibshirani R. (1990), "Generalized Additive Models", *Chapman and Hall.*

[16] Hayat A., Mishra S. (2010), "Federal Reserve Monetary Policy and the Non-linearity of the Taylor Rule", *Economic Modelling*, 27(5), 1292-1301.

[17] Judd J.P. and Rudebusch G. (1998), "Taylor's Rule and the Fed: 1970-1997", *FRBSF Economic Review*, 3.

[18] Kai L., Lonning I. (2006), "Simple Monetary Policymaking Without the Output Gap", *Journal of Money, Credit and Banking*, 38(6), 1619-1640.

[19] Kim Y-J. and Gu C. (2004), "Smoothing Spline Gaussian Regression: More Scalable Computation via Efficient Approximation", *Journal of the Royal Statistical Society*, 66(2), 337-356.

[20] Kim D.H., Osborn D.R., Sensier M. (2005), "Nonlinearity in the Fed's Monetary Policy Rule", *Journal of Applied Econometrics*, 20(5), 621-639.

[21] Kirman A.P. (1992), "Whom or What does the Representative Individual Represent?", *Journal of Economic Perspectives*, 6(2), 117-136.

[22] Klose J. (2011), "Asymmetric Taylor Reaction Functions of the ECB: An Approach Depending on the State of the Economy", *North American Journal of Economics and Finance*, 22(2), 149-163.

[23] Li Q., Racine J.S. (2007), "Nonparametric Econometrics. Theory and Practice", *Princeton University Press.*

[24] Owyang M., Ramey G. (2004), "Regime Switching and Monetary Policy Measurement", *Journal of Monetary Economics*, 51(8), 1577-1597.

[25] Rotemberg J. and Woodford M. (1999), "Interest Rate Rules in an Estimated Sticky Price Model", in *Monetary Policy Rules* edited by John Taylor.

[26] Rudebusch G. (2002), "Term Structure Evidence on Interest Rate Smoothing and Monetary Policy Inertia", *Journal of Monetary Economics*, 49(6), 1161-1187.

[27] Rudebusch G. (2006), "Monetary Policy Inertia: Fact or Fiction?", *International Journal of Central Banking*, 2(4), 85-135.

[28] Sims C., Zha T. (2006), "Were There Regime Switches in U.S. Monetary Policy?", *American Economic Review*, 96(1), 54-81.

[29] Taylor J. (1993), "Discretion versus Policy Rules in Practice", *Carnegie-Rochester Conference Series on Public Policy*, 39.

[30] Taylor J., Williams J. (2010), "Simple and Robust Rules for Monetary Policy", *Federal Reserve Bank of San Francisco Working Paper*, 10.

[31] Walsh C. (2003), "Speed Limit Policies: The Output Gap and Optimal Monetary Policy", *American Economic Review*, 93(1), 265-278.

[32] Wood S.N. (2006), "Generalized Additive Models. An introduction with R", *Chapman and Hall/CRC.*

[33] Woodford M. (2003), "Interest and Prices: Foundations of a Theory of Monetary Policy", *Princeton University Press.*

9 Appendix

9.1 Proofs of the Theoretical framework

Proof of Proposition 1. *Proof:*

1. *Point 1 is obtained by replacing the constraint, $\psi(E_t\pi_{t+1}, x_t)+u_t$ in the Central Bank loss function $h(x_t, \pi_t)$ and differentiating with respect to x_t.*
2. *It is straightforward from the FOC to get the desired result by totally differentiating with respect to $E_t x_t$ and $E_t\pi_t$.*
3. *The FOC is of the form $\rho(E_t x_{t+1}, E_t\pi_{t+1}, i_t) = 0$. Since $\frac{\partial\rho(.,.,.)}{\partial E_t x_{t+1}}$, $\frac{\partial\rho(.,.,.)}{\partial E_t\pi_{t+1}}$, $\frac{\partial\rho(.,.,.)}{\partial i_t}$ $\in \mathbb{R}$. This ensures the existence of $\frac{\partial\Omega(.,.)}{\partial E_t x_{t+1}}$ and $\frac{\partial\Omega(.,.)}{\partial E_t\pi_{t+1}}$ and therefore the existence of $\Omega(E_t\pi_{t+1}, E_t x_{t+1})$.*

Proof of Proposition 2. *Proof:*

1. *From the result of Point 2 in Proposition 1. We compute,*

$$dE_0x_0 = \eta_{i_0-E_0\pi_1}(E_0x_1, i_0 - E_0\pi_1)di_0 - \eta_{i_0-E_0\pi_1}(E_0x_1, i_0 - E_0\pi_1)dE_0\pi_1$$

and

$$\begin{aligned} dE_0\pi_0 &= \psi_{E_0\pi_1}(E_0\pi_1, x_0)dE_0\pi_1 + \psi_{x_0}(E_0\pi_1, x_0)\eta_{i_0-E_0\pi_1}(E_0x_1, i_0 - E_0\pi_1)di_0 \\ &\quad -\psi_{x_0}(E_0\pi_1, x_0)\eta_{i_0-E_0\pi_1}(E_0x_1, i_0 - E_0\pi_1)dE_0\pi_1 \end{aligned}$$

By assuming $dE_0x_1 = 0$ and after some simple algebra, totally differentiating the FOC yields the solution.

2. *Same procedure as in the previous point but this time we assume $dE_0\pi_1 = 0$. We compute,*

$$dE_0x_0 = \eta_{E_0x_1}(E_0x_1, i_0 - E_0\pi_1)dE_0x_1 + \eta_{i_0-E_0\pi_1}(E_0x_1, i_0 - E_0\pi_1)di_0$$

and

$$\begin{aligned} dE_0\pi_0 = \psi_{x_0}(E_0\pi_1, x_0)\eta_{E_0x_1}(E_0x_1, i_0 - E_0\pi_1)dE_0x_1 + \\ \psi_{x_0}(E_0\pi_1, x_0)\eta_{i_0-E_0\pi_1}(E_0x_1, i_0 - E_0\pi_1)di_0 \end{aligned}$$

3. *From the result in point 1, we differentiate with respect to $E_0\pi_1$ and after some calculations we get the desired expression.*
4. *Points 4. and 5. are solved as in the previous point but by differentiating with respect to the concerned variables in each point instead.*

9.2 Data description

Interest rates:

Australia: The key policy rate used in the regressions is the monthly average interbank overnight rate (the cash rate).

Canada: Since May 2001, the key policy rate used is the Bank of Canada target for the overnight rate. Prior to May 2001, the Bank rate has been used in the regressions.

Japan: The policy rate used is the target uncollateralized middle monthly average rate.

New Zealand: The policy rate used is the official overnight target rate (the cash rate).

Norway: As a policy rate we have used the overnight market lending rate since March 1987. Prior to this date, the discount rate has been used as a policy rate.

Sweden: Since June 1994, the policy rate used is the repo rate. Prior to this date we have used the discount rate.

Switzerland: Since January 2000, the policy rate used is the target for the 3 month Libor rate on the CHF. Prior to 2000, we have used the discount rate.

USA: The interest rate used is the monthly average effective federal funds rate.

All interest rates are reported for the month following the release of the inflation and real GDP forecasts within the same quarter in order to avoid a possible endogeneity problem.

Forecast variables:

As explanatory variables, we have used the quarterly OECD forecasts of inflation and real GDP for the current year which are downloaded from the Datastream forecast series. The variables are constructed with a one-year fixed horizon for the current year with respect to the corresponding quarter of the previous year. We also use Consensus Economics Forecasts of inflation and real GDP growth with a one-year fixed horizon on a monthly basis. In the baseline regressions, the nonlinear specifications are smooth functions of the inflation and real GDP growth forecasts. In the robustness analysis, we also perform the regressions with a Hodrick-Prescott (H-P) detrended change in the output gap. A smoothing parameter of 1600 and 14400 has been used correspondingly in the filtering procedure to account for the quarterly and monthly frequencies of the data.

Table 1: Summary statistics, OECD forecasts

DEPENDENT AND EXPLANATORY VARIABLES	Obs.	Mean	Std. deviation	Min	Max
Australia					
Policy interest rate (%)	138	8.711	4.358	3.00	19.39
Inflation forecasts (%)	138	5.228	3.618	-0.34	14.23
Real GDP growth forecasts (%)	138	3.216	1.978	-3.07	8.36
Output gap change forecasts (%)	138	2.73e-9	1.776	-5.84	5.03
Canada					
Policy interest rate (%)	138	7.136	4.248	0.25	20.04
Inflation forecasts (%)	138	4.030	3.169	-0.89	12.68
Real GDP growth forecasts (%)	138	2.683	2.276	-3.71	6.55
Output gap change forecasts (%)	138	4.90e-9	1.819	-5.98	3.39
Japan					
Policy interest rate (%)	138	2.529	2.421	0.10	9.00
Inflation forecasts (%)	138	1.587	2.544	-2.30	10.50
Real GDP growth forecasts (%)	138	2.506	2.730	-8.69	9.27
Output gap change forecasts (%)	138	1.20e-8	1.927	-8.56	4.74
New Zealand					
Policy interest rate (%)	138	10.133	5.075	2.50	28.20
Inflation forecasts (%)	138	6.308	5.780	-0.51	18.93
Real GDP growth forecasts (%)	138	2.197	3.319	-12.89	14.72
Output gap change forecasts (%)	138	1.04e-9	2.811	-11.82	11.19
Norway					
Policy interest rate (%)	138	7.609	2.569	2.25	13.80
Inflation forecasts (%)	138	4.562	3.424	-1.30	14.63
Real GDP growth forecasts (%)	138	2.846	2.243	-2.39	8.29
Output gap change forecasts (%)	138	3.00e-9	1.757	-4.19	4.87
Sweden					
Policy interest rate (%)	138	6.278	3.188	0.25	12.00
Inflation forecasts (%)	138	4.691	4.116	-1.12	14.73
Real GDP growth forecasts (%)	138	2.013	2.382	-6.61	5.76
Output gap change forecasts (%)	138	9.47e-9	1.905	-7.18	4.56
Switzerland					
Policy interest rate (%)	138	2.728	1.908	0.25	7.00
Inflation forecasts (%)	138	2.110	1.850	-0.97	7.16
Real GDP growth forecasts (%)	138	1.734	1.797	-3.32	6.17
Output gap change forecasts (%)	138	4.31e-7	1.547	-4.76	3.91
USA					
Policy interest rate (%)	138	6.031	3.834	0.12	19.10
Inflation forecasts (%)	138	4.085	2.900	-1.60	14.41
Real GDP growth forecasts (%)	138	2.868	2.211	-4.11	8.48
Output gap change forecasts (%)	138	-1.21e-9	1.748	-5.08	4.93

Note: The policy interest rates are taken from the Central Bank's official websites and Datastream. The inflation and real GDP growth forecasts are performed by the OECD and are downloaded from Datastream. The forecasts for the change in the output gap are computed by the authors using an H-P filter.

Table 2: Unit root tests, OECD forecasts

Variables by country	ADF Z(t)	PP Z(t)	KPSS	Integration order
Australia				
Policy interest rate	-1.617*	-1.957	1.080***	I(1)
	(0.054)	(0.306)		
Inflation forecasts	-3.260***	-2.659*	1.350***	I(0)
	(0.001)	(0.082)		
Real GDP growth forecasts	-6.033***	-4.515***	0.071	I(0)
	(0.000)	(0.000)		
Output gap change forecasts	-7.193***	-4.953***	0.021	I(0)
	(0.000)	(0.000)		
Canada				
Policy interest rate	-1.656**	-1.432	1.450***	I(1)
	(0.050)	(0.567)		
Inflation forecasts	-1.785**	-1.527	1.250***	I(1)
	(0.038)	(0.520)		
Real GDP growth forecasts	-4.848***	-3.782***	0.118	I(0)
	(0.000)	(0.003)		
Output gap change forecasts	-7.013***	-4.377***	0.023	I(0)
	(0.000)	(0.000)		
Japan				
Policy interest rate	-1.742**	-1.697	1.530***	I(1)
	(0.042)	(0.433)		
Inflation forecasts	-3.573***	-3.342**	1.310***	I(0)
	(0.000)	(0.013)		
Real GDP growth forecasts	-4.509***	-3.555***	0.976***	I(0)
	(0.000)	(0.007)		
Output gap change forecasts	-8.302***	-5.048***	0.022	I(0)
	(0.000)	(0.000)		
New Zealand				
Policy interest rate	-1.336*	-1.881	1.070***	I(1)
	(0.092)	(0.341)		
Inflation forecasts	-2.355***	-2.388	1.350***	I(1)
	(0.010)	(0.145)		
Real GDP growth forecasts	-5.251***	-6.428***	0.234	I(0)
	(0.000)	(0.000)		
Output gap change forecasts	-8.309***	-7.885***	0.024	I(0)
	(0.000)	(0.000)		
Norway				
Policy interest rate	-1.947**	-1.888	0.748***	I(1)
	(0.027)	(0.338)		
Inflation forecasts	-2.329**	-2.013	1.350***	I(1)
	(0.011)	(0.281)		
Real GDP growth forecasts	-4.174***	-5.547***	0.377*	I(0)
	(0.000)	(0.000)		
Output gap change forecasts	-6.327***	-7.574***	0.025	I(0)
	(0.000)	(0.000)		
Sweden				
Policy interest rate	-1.446*	-1.187	1.470***	I(1)
	(0.075)	(0.679)		
Inflation forecasts	-2.636***	-1.964	1.450***	I(1)
	(0.005)	(0.303)		
Real GDP growth forecasts	-5.903***	-3.572***	0.112	I(0)
	(0.000)	(0.006)		
Output gap change forecasts	-7.913***	-4.151***	0.022	I(0)
	(0.000)	(0.001)		
Switzerland				
Policy interest rate	-2.032**	-1.810	0.619**	I(1)
	(0.022)	(0.375)		
Inflation forecasts	-2.305**	-2.247	0.725**	I(1)
	(0.011)	(0.190)		
Real GDP growth forecasts	-5.297***	-4.013***	0.050	I(0)
	(0.000)	(0.001)		
Output gap change forecasts	-6.594***	-4.421***	0.022	I(0)
	(0.000)	(0.000)		
USA				
Policy interest rate	-1.522*	-1.635	1.220***	I(1)
	(0.065)	(0.465)		
Inflation forecasts	-2.388***	-1.846	0.960***	I(1)
	(0.009)	(0.358)		
Real GDP growth forecasts	-4.771***	-3.770***	0.232	I(0)
	(0.000)	(0.003)		
Output gap change forecasts	-6.794***	-4.493***	0.024	I(0)
	(0.000)	(0.000)		

Note: The ADF Z(t) and PP Z(t) refer to the Augmented Dickey-Fuller and Phillips-Perron tests for unit root in the variables. A statistically significant test shows evidence against the null hypothesis of unit root. For the ADF test 3 lags of the difference in the variables have been used, while the number of lags used in the Phillips-Perron test are determined automatically based on Newey-West bandwidth selection. The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test reports the statistic for testing the null hypothesis of level stationarity of the series based on Newey-West automatic bandwidth selection. A statistically significant test shows evidence against the hypothesis of stationarity. The integration order is determined using the ADF, PP and KPSS tests on the differenced variables. MacKinnon approximate p-values are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 3: Summary statistics, Consensus forecasts

DEPENDENT AND EXPLANATORY VARIABLES	Obs.	Mean	Std. deviation	Min	Max
Canada					
Policy interest rate (%)	244	4.828	2.830	0.25	13.90
Inflation forecasts (%)	244	2.208	1.070	0.44	6.07
Real GDP growth forecasts (%)	244	2.546	1.038	-1.22	3.89
Output gap change forecasts (%)	244	1.15e-9	0.653	-2.18	2.07
Japan					
Policy interest rate (%)	244	1.160	1.701	0.10	6.00
Inflation forecasts (%)	244	0.475	1.045	-1.10	3.09
Real GDP growth forecasts (%)	244	1.426	1.511	-4.68	4.95
Output gap change forecasts (%)	244	-1.13e-16	0.876	-4.32	2.86
Norway					
Policy interest rate (%)	244	6.938	2.408	2.25	17.00
Inflation forecasts (%)	244	2.512	0.879	0.95	4.97
Real GDP growth forecasts (%)	244	2.420	0.970	-0.70	3.90
Output gap change forecasts (%)	244	-5.64e-10	0.621	-2.29	1.66
Sweden					
Policy interest rate (%)	244	4.808	2.865	0.25	12.00
Inflation forecasts (%)	244	2.583	2.007	-0.19	9.42
Real GDP growth forecasts (%)	244	1.951	1.281	-2.40	3.95
Output gap change forecasts (%)	244	-1.34e-9	0.695	-2.96	2.46
Switzerland					
Policy interest rate (%)	244	2.455	2.059	0.25	7.00
Inflation forecasts (%)	244	1.695	1.165	-0.16	4.59
Real GDP growth forecasts (%)	244	1.574	0.721	-1.52	2.85
Output gap change forecasts (%)	244	1.38e-9	0.487	-2.01	1.65
USA					
Policy interest rate (%)	244	3.991	2.068	0.13	8.25
Inflation forecasts (%)	244	2.700	0.894	-0.66	5.20
Real GDP growth forecasts (%)	244	2.500	1.151	-2.06	4.62
Output gap change forecasts (%)	244	2.74e-9	0.710	-2.78	2.41

Note: The policy interest rates are taken from the Central Bank's official websites and Datastream. The inflation and real GDP growth forecasts are performed by Consensus Economics and are provided by Datastream and the Swiss National Bank. The forecasts for the change in the output gap are computed by the authors using an H-P filter.

Table 4: Unit root tests, Consensus forecasts

Variables by country	ADF Z(t)	PP Z(t)	KPSS	Integration order
Canada				
Policy interest rate	-2.750***	-2.223	1.430***	I(1)
	(0.003)	(0.198)		
Inflation forecasts	-3.216***	-2.638*	0.683**	I(0)
	(0.000)	(0.085)		
Real GDP growth forecasts	-3.976***	-2.828*	0.280	I(0)
	(0.002)	(0.054)		
Output gap change forecasts	-6.069***	-3.714***	0.020	I(0)
	(0.000)	(0.004)		
Japan				
Policy interest rate	-2.956***	-1.748	1.300***	I(1)
	(0.002)	(0.407)		
Inflation forecasts	-2.055**	-1.437	1.260***	I(1)
	(0.021)	(0.564)		
Real GDP growth forecasts	-4.258***	-2.886**	0.580**	I(0)
	(0.000)	(0.047)		
Output gap change forecasts	-6.652***	-3.639***	0.022	I(0)
	(0.000)	(0.005)		
Norway				
Policy interest rate	-1.930**	-1.873	1.150***	I(1)
	(0.027)	(0.345)		
Inflation forecasts	-3.330***	-2.899**	1.170***	I(0)
	(0.001)	(0.045)		
Real GDP growth forecasts	-4.203***	-2.616*	0.155	I(0)
	(0.000)	(0.090)		
Output gap change forecasts	-6.507***	-3.556***	0.021	I(0)
	(0.000)	(0.007)		
Sweden				
Policy interest rate	-2.431***	-1.525	1.700***	I(1)
	(0.008)	(0.521)		
Inflation forecasts	-2.733***	-1.840	1.090***	I(1)
	(0.003)	(0.361)		
Real GDP growth forecasts	-3.750***	-2.299	0.492**	I(0)
	(0.000)	(0.172)		
Output gap change forecasts	-7.064***	-3.355**	0.023	I(0)
	(0.000)	(0.013)		
Switzerland				
Policy interest rate	-1.886**	-1.493	1.210***	I(1)
	(0.030)	(0.537)		
Inflation forecasts	-2.003**	-1.290	1.450***	I(1)
	(0.023)	(0.634)		
Real GDP growth forecasts	-5.051***	-2.923**	0.184	I(0)
	(0.000)	(0.044)		
Output gap change forecasts	-6.701***	-3.384**	0.023	I(0)
	(0.000)	(0.012)		
USA				
Policy interest rate	-2.234**	-1.461	0.809***	I(1)
	(0.013)	(0.553)		
Inflation forecasts	-3.083***	-2.843*	1.170***	I(0)
	(0.001)	(0.053)		
Real GDP growth forecasts	-3.729***	-2.737*	0.221	I(0)
	(0.000)	(0.068)		
Output gap change forecasts	-5.918***	-3.652***	0.023	I(0)
	(0.000)	(0.005)		

Note: The ADF Z(t) and PP Z(t) refer to the Augmented Dickey-Fuller and Phillips-Perron tests for unit root in the variables. A statistically significant test shows evidence against the null hypothesis of unit root. For the ADF test 3 lags of the difference in the variables have been used, while the number of lags used in the Phillips-Perron test are determined automatically based on Newey-West bandwidth selection. The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test reports the statistic for testing the null hypothesis of level stationarity of the series based on Newey-West automatic bandwidth selection. A statistically significant test shows evidence against the hypothesis of stationarity. The integration order is determined using the ADF, PP and KPSS tests on the differenced variables. MacKinnon approximate p-values are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

9.3 Estimation tables, OECD forecasts

Table 5: Bivariate GAM, output growth

	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switzerland	USA
constant	0.087107	0.07136	0.0252935	0.101326	0.076093	0.0627783	0.0272826	0.060312
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t\pi_{t+1}, E_t x_{t+1})$	9.84	16.80	14.37	5.60	10.10	21.84	4.39	15.25
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(5.73e-09)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
Obs.	138	138	138	138	138	138	138	138
Adj. R^2	0.649	0.802	0.826	0.535	0.395	0.793	0.592	0.716
Dev. Exp.	67.4%	82.6%	84.4%	55.4%	43.9%	82.6%	60.5%	74.7%
AIC	-605.1	-685.0	-860.4	-528.4	-675.6	-754.1	-817.6	-664.8

Table 6: Univariate GAM with no interactions, output growth

	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switzerland	USA
constant	0.087107	0.07136	0.0252935	0.101326	0.076093	0.0627783	0.027283	0.060312
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t\pi_{t+1})$	3.74	3.56	4.84	3.22	2.53	2.53	1.23	1.00
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(1.01e-07)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t x_{t+1})$	2.36	4.51	1.00	1.00	1.21	1.74	1.00	1.84
	(0.0200)**	(0.00419)***	(0.000196)***	(0.242)	(0.543)	(0.240)	(0.786)	(0.00759)***
Obs.	138	138	138	138	138	138	138	138
Adj. R^2	0.624	0.757	0.77	0.552	0.234	0.688	0.578	0.606
Dev. Exp.	64.1%	77.1%	77.9%	56.6%	25.5%	69.7%	58.5%	61.5%
AIC	-599.2	-664.6	-829.3	-534.7	-649.1	-712.9	-814.9	-631.4

Table 7: Univariate GAM with interactions, output growth

	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switzerland	USA
constant	0.087107	0.07136	0.0252935	0.101326	0.076093	0.0627783	0.0272826	0.060312
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t\pi_{t+1})$	4.36	2.79	3.51	2.99	2.09	6.23	1.00	3.77
	(1.95e-08)***	(1.23e-14)***	(7.22e-05)***	(6.10e-11)***	(1.35e-05)***	(<2e-16)***	(1.43e-10)***	(<2e-16)***
$s(E_t x_{t+1})$	1.00	1.00	7.47	1.00	1.00	6.38	2.04	4.72
	(1.84e-07)***	(4.60e-10)***	(7.43e-06)***	(0.000711)***	(0.0185)**	(9.83e-16)***	(0.01316)**	(1.23e-06)***
$s(E_t\pi_{t+1} \times t)$	3.06	5.98	7.68	2.06	1.51	7.25	4.69	1.00
	(4.57e-08)***	(0.00102)***	(0.00013)***	(1.27e-06)***	(0.2291)	(2.47e-06)***	(2.41e-05)***	(0.00193)***
$s(E_t x_{t+1} \times t)$	6.15	4.69	6.16	2.26	3.08	7.49	4.51	7.48
	(1.23e-08)***	(4.03e-07)***	(5.20e-07)***	(0.002238)***	(1.65e-05)***	(<2e-16)***	(0.00562)***	(6.39e-06)***
$s(E_t\pi_{t+1} \times E_t x_{t+1})$	2.90	3.77	3.65	1.00	5.44	7.54	1.00	4.82
	(0.000411)***	(4.15e-09)***	(5.13e-06)***	(0.006559)***	(4.28e-05)***	(6.57e-16)***	(8.95e-05)***	(5.02e-06)***
Obs.	138	138	138	138	138	138	138	138
Adj. R^2	0.767	0.845	0.883	0.652	0.471	0.917	0.683	0.783
Dev. Exp.	79.7%	86.5%	90.7%	67.6%	52.2%	93.8%	71.4%	81.8%
AIC	-655.1	-717.4	-903.5	-565.0	-691.5	-871.4	-844.3	-696.8

Note: The tables report the estimated degrees of freedom of the smooth functions from the GAM specifications. The latter are functions of economic fundamentals, $E_t\pi_{t+1}$ and $E_t x_{t+1}$. The last specification also includes interactions between the inflation and real output growth forecasts as well as interactions between the economic fundamentals and time. P-values are reported in parentheses. Superscripts (***) , (**) and (*) indicate significance level at 99%, 95% and 90% respectively.

Table 8: Root Forecast Mean Squared Error, output growth

Forecasting models	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switzerland	USA
Bivariate GAM 1	0.01527523	***0.01776179***	0.003681934	0.0441975	***0.01441452***	0.01977281	***0.005763357***	0.02749656
Bivariate GAM 2	0.02214266	0.0263695	***0.01074736***	0.03015509	0.0249533	0.02110059	***0.01167997***	0.02327633
Univariate GAM 1	0.01403327	0.02445593	***0.00052674***	0.04265023	0.03252064	***0.01646734***	0.007334405	0.02865517
Univariate GAM 2	0.02268192	0.02709358	0.0117875	***0.02977952***	0.02787892	0.02120447	0.01202762	0.02500962
Augmented GAM 1	***0.003082172***	0.02193136	0.00242419	***0.03580241***	0.04023434	0.0313614	0.01113345	***0.02025786***
Augmented GAM 2	***0.01776811***	***0.01899115***	0.01546564	0.03712306	***0.02087202***	***0.0130202***	0.01367746	***0.01748774***
Linear model 1	0.01740731	0.02259563	0.008553875	0.04687572	0.04435981	0.02393431	0.007334405	0.02685988
Linear model 2	0.02522371	0.02833424	0.01398219	0.03105879	0.02953699	0.02521325	0.01203019	0.02424018
Forecast Gain 1	31.897	1.618	263.714	1.714	9.471	2.112	1.619	1.758
Forecast Gain 2	2.015	2.226	1.693	1.088	2.003	3.750	1.061	1.921

Note: The table reports the out-of-sample forecasting performance of the semi-parametric and linear models. For each model the first row displays the RFMSE of the forecasts for 2010, while the second row indicates the corresponding RFMSE for the rolling window forecasts. The Univariate GAM refers to the model without interactions, while the Augmented GAM corresponds to the model with the relevant interaction terms. The best forecasting performance is reported in bold characters. The forecast gain is defined as the ratio between the forecast mean squared error of the linear model and the one of the best model for each country as indicated in bold characters.

Table 9: Bivariate GAM, output gap change

	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switzerland	USA
constant	0.087107	0.07136	0.0252935	0.101326	0.076093	0.0627783	0.0272826	0.060312
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t\pi_{t+1}, E_t x_{t+1})$	9.51	17.18	9.73	12.16	6.22	7.12	2.49	18.59
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(2.55e-06)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
Obs.	138	138	138	138	138	138	138	138
Adj. R^2	0.643	0.798	0.759	0.603	0.267	0.709	0.579	0.701
Dev. Exp.	66.8%	82.3%	77.6%	63.8%	30%	72.4%	58.7%	74.1%
AIC	-603.1	-682.1	-819.4	-544.2	-652.8	-719.7	-815.1	-655.0

Table 10: Univariate GAM with no interactions, output gap change

	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switzerland	USA
constant	0.087107	0.07136	0.0252935	0.101326	0.076093	0.0627783	0.027283	0.060312
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t\pi_{t+1})$	3.66	3.63	3.86	3.22	2.57	2.57	1.02	1.72
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(6.43e-07)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t x_{t+1})$	2.99	1.00	1.00	1.00	1.57	2.20	1.00	2.84
	(0.0833)*	(0.00967)***	(0.484)	(0.963)	(0.105)	(0.0246)**	(0.971)	(0.333)
Obs.	138	138	138	138	138	138	138	138
Adj. R^2	0.616	0.739	0.74	0.547	0.253	0.704	0.576	0.594
Dev. Exp.	63.5%	74.8%	74.9%	56.1%	27.5%	71.4%	58.3%	60.7%
AIC	-596.0	-658.0	-813.6	-533.3	-652.2	-719.7	-814.6	-625.4

Table 11: Univariate GAM with interactions, output gap change

	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switzerland	USA
constant	0.087107	0.07136	0.0252935	0.101326	0.076093	0.0627783	0.0272826	0.060312
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t\pi_{t+1})$	5.24	3.04	2.85	3.01	1.00	1.02	1.00	6.40
	(2.92e-14)***	(<2e-16)***	(<2e-16)***	(5.46e-15)***	(0.108947)	(1.19e-11)***	(8.6e-12)***	(9.45e-14)***
$s(E_t x_{t+1})$	4.94	6.20	4.25	1.00	3.78	5.97	1.00	1.34
	(0.00156)***	(8.83e-05)***	(1.52e-09)***	(0.239539)	(0.001056)***	(2.20e-09)***	(0.62201)	(0.537264)
$s(E_t\pi_{t+1} \times t)$	3.95	3.62	2.24	2.18	1.09	7.24	4.36	2.01
	(7.75e-05)***	(0.00672)***	(0.0473)**	(0.000138)***	(0.006105)***	(2.44e-09)***	(0.00109)***	(8.96e-05)***
$s(E_t x_{t+1} \times t)$	5.00	3.58	6.05	1.18	3.56	6.98	2.93	3.23
	(0.00642)***	(4.42e-06)***	(2.87e-08)***	(0.730603)	(0.000167)***	(7.96e-13)***	(0.12451)	(0.005344)***
$s(E_t\pi_{t+1} \times E_t x_{t+1})$	3.21	1.00	1.00	1.08	2.81	3.70	1.00	5.27
	(0.01112)**	(0.40874)	(0.0199)**	(0.326946)	(0.078148)*	(9.10e-07)***	(0.63890)	(0.000153)***
Obs.	138	138	138	138	138	138	138	138
Adj. R^2	0.744	0.827	0.836	0.615	0.358	0.849	0.647	0.765
Dev. Exp.	78.5%	84.9%	85.5%	63.8%	41.5%	87.6%	67.4%	79.6%
AIC	-637.8	-703.0	-866.4	-551.6	-665.5	-795.0	-832.1	-688.3

Note: The tables report the estimated degrees of freedom of the smooth functions from the GAM specifications. The latter are functions of economic fundamentals, $E_t\pi_{t+1}$ and $E_t x_{t+1}$. The last specification also includes interactions between the inflation and output gap change forecasts as well as interactions between the economic fundamentals and time. P-values are reported in parentheses. Superscripts (***) , (**) and (*) indicate significance level at 99%, 95% and 90% respectively.

Table 12: Root Forecast Mean Squared Error, output gap change

Forecasting models	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switzerland	USA
Bivariate GAM 1	0.01740715	0.04857935	0.005852322	0.0365132	0.03680485	0.01840426	***0.006863292***	0.03894011
Bivariate GAM 2	0.0228735	0.02662497	0.01390077	***0.0283817***	***0.02526091***	0.02222503	***0.01182536***	0.02584867
Univariate GAM 1	0.01607842	0.05017287	0.004848005	0.03726073	0.03753655	0.01677202	0.00735254	0.0351024
Univariate GAM 2	0.02343475	0.02705798	0.01444524	0.03044677	0.02552442	0.02099072	0.0119321	0.02643879
Augmented GAM 1	***0.008417817***	***0.02378592***	***0.003250535***	***0.01522484***	***0.02302403***	***0.01526226***	0.009367417	***0.01864445***
Augmented GAM 2	***0.01632975***	***0.019759***	***0.01126629***	0.0383722	0.02627061	***0.01668107***	0.01308924	***0.01434367***
Linear model 1	0.01853351	0.04695009	0.003860677	0.04117923	0.04133999	0.02523351	0.007654218	0.03303874
Linear model 2	0.02541442	0.02826132	0.01613256	0.03145757	0.02707373	0.02551477	0.01193628	0.02561315
Forecast Gain 1	4.850	3.896	1.410	7.316	3.224	2.733	1.240	3.140
Forecast Gain 2	2.422	2.046	2.050	1.228	1.149	2.340	1.019	3.189

Note: The table reports the out-of-sample forecasting performance of the semi-parametric and linear models. For each model the first row displays the RFMSE of the forecasts for 2010, while the second row indicates the corresponding RFMSE for the rolling window forecasts. The Univariate GAM refers to the model without interactions, while the Augmented GAM corresponds to the model with the relevant interaction terms. The best forecasting performance is reported in bold characters. The forecast gain is defined as the ratio between the forecast mean squared error of the linear model and the one of the best model for each country as indicated in bold characters.

9.4 Estimation tables, Consensus Economics Forecasts

Table 13: Bivariate GAM, CEF output growth

	Canada	Japan	Norway	Sweden	Switzerland	USA
constant	0.048279	0.0115984	0.0693832	0.0480811	0.0245492	0.0399078
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t\pi_{t+1}, E_t x_{t+1})$	17.56	24.63	20.83	21.03	25.70	21.42
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
Obs.	244	244	244	244	244	244
Adj. R^2	0.789	0.951	0.725	0.816	0.955	0.632
Dev. Exp.	80.4%	95.6%	74.9%	83.2%	96%	66.4%
AIC	-1407.2	-2004.5	-1418.4	-1430.9	-1933.7	-1420.5

Table 14: Univariate GAM with no interactions, CEF output growth

	Canada	Japan	Norway	Sweden	Switzerland	USA
constant	0.0482791	0.0115984	0.069383	0.0480811	0.0245492	0.0399078
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t\pi_{t+1})$	3.10	8.63	1.00	4.15	7.64	7.44
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t x_{t+1})$	5.53	8.74	7.71	1.75	7.24	3.89
	(<2e-16)***	(7.6e-08)***	(<2e-16)***	(0.361)	(5.96e-08)***	(9.02e-05)***
Obs.	244	244	244	244	244	244
Adj. R^2	0.757	0.945	0.658	0.752	0.911	0.557
Dev. Exp.	76.6%	94.9%	67.1%	75.8%	91.6%	57.7%
AIC	-1380.9	-1983.7	-1376.6	-1373.1	-1774.7	-1384.7

Table 15: Univariate GAM with interactions, CEF output growth

	Canada	Japan	Norway	Sweden	Switzerland	USA
constant	0.0482791	0.0115984	0.0693832	0.0480811	0.0245492	0.0399078
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t\pi_{t+1})$	2.35	8.92	3.02	8.25	8.30	3.29
	(5.86e-08)***	(<2e-16)***	(6.54e-05)***	(<2e-16)***	(9.16e-15)***	(8.85e-15)***
$s(E_t x_{t+1})$	6.48	6.72	5.01	3.99	8.47	6.62
	(2.14e-12)***	(2.19e-09)***	(0.0524)*	(0.00148)***	(7.90e-11)**	(0.0259)**
$s(E_t\pi_{t+1} \times t)$	6.66	7.59	8.50	8.13	2.70	8.72
	(0.001087)***	(< 2e-16)***	(5.50e-08)***	(< 2e-16)***	(0.356)	(< 2e-16)***
$s(E_t x_{t+1} \times t)$	7.78	6.35	5.89	8.70	5.72	8.57
	(0.000365)***	(9.79e-11)***	(3.37e-06)***	(2.53e-06)***	(4.46e-05)***	(<2e-16)***
$s(E_t\pi_{t+1} \times E_t x_{t+1})$	5.27	2.11	4.73	7.26	7.41	6.33
	(0.065000)*	(0.0781)*	(0.0406)**	(0.00214)***	(< 2e-16)***	(0.0240)**
Obs.	244	244	244	244	244	244
Adj. R^2	0.902	0.986	0.815	0.95	0.958	0.867
Dev. Exp.	91.4%	98.8%	83.6%	95.7%	96.4%	88.6%
AIC	-1584.7	-2298.9	-1509.3	-1734.9	-1945.4	-1659.1

Note: The tables report the estimated degrees of freedom of the smooth functions from the GAM specifications. The latter are functions of economic fundamentals, $E_t\pi_{t+1}$ and $E_t x_{t+1}$. The last specification also includes interactions between the inflation and real output growth forecasts as well as interactions between the economic fundamentals and time. P-values are reported in parentheses. Superscripts (***) , (**) and (*) indicate significance level at 99%, 95% and 90% respectively.

Table 16: Root Forecast Mean Squared Error, CEF output growth

Forecasting models	Canada	Japan	Norway	Sweden	Switzerland	USA
Bivariate GAM 1	0.03089823	***0.00064957 8***	0.02571209	0.02560193	0.008049294	***0.01818179***
Bivariate GAM 2	0.0131712	0.004257767	0.01728486	0.01770866	***0.00563068***	***0.01317502***
Univariate GAM 1	0.03224988	0.005687875	***0.02440984***	0.02072213	0.007309076	0.02036682
Univariate GAM 2	0.01369091	0.005157137	0.01805622	0.01598655	0.006604679	0.01461539
Augmented GAM 1	0.04348549	0.002547621	0.02771728	***0.01663119***	0.0235398	0.0214123
Augmented GAM 2	***0.01066325***	***0.002430394***	***0.01295781***	***0.01133677***	0.00925362	0.01537729
Linear model 1	***0.02582959***	0.01103968	0.03037086	0.02710942	***0.000662576***	0.02302397
Linear model 2	0.01504188	0.009330098	0.0256323	0.01867243	0.008171749	0.01801428
Forecast Gain 1	0.699	288.836	1.548	2.657	0.008	1.604
Forecast Gain 2	1.990	14.737	3.913	2.713	2.106	1.870

Note: The table reports the out-of-sample forecasting performance of the semi-parametric and linear models. For each model the first row displays the RFMSE of the forecasts from December 2009 to March 2010, while the second row indicates the corresponding RFMSE for the rolling window forecasts. The Univariate GAM refers to the model without interactions, while the Augmented GAM corresponds to the model with the relevant interaction terms. The best forecasting performance is reported in bold characters. The forecast gain is defined as the ratio between the forecast mean squared error of the linear model and the one of the best semi-parametric model for each country as indicated in bold characters.

Table 17: Bivariate GAM, CEF output gap change

	Canada	Japan	Norway	Sweden	Switzerland	USA
constant	0.048279	0.0115984	0.0693832	0.0480811	0.0245492	0.0399078
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t\pi_{t+1}, E_t x_{t+1})$	17.99	18.01	10.65	10.89	21.57	20.08
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
Obs.	244	244	244	244	244	244
Adj. R^2	0.744	0.955	0.592	0.772	0.926	0.613
Dev. Exp.	76.3%	95.9%	61%	78.2%	93.3%	64.5%
AIC	-1359.8	-2033.4	-1331.6	-1388.1	-1814.5	-1409.6

Table 18: Univariate GAM with no interactions, CEF output gap change

	Canada	Japan	Norway	Sweden	Switzerland	USA
constant	0.0482791	0.0115984	0.069383	0.0480811	0.0245492	0.0399078
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t\pi_{t+1})$	3.20	8.25	1.89	5.02	7.83	7.68
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t x_{t+1})$	6.00	5.28	2.88	2.57	3.68	7.42
	(1.15e-10)***	(8.15e-05)***	(3.35e-06)***	(0.0562)*	(0.000369)***	(0.000158)***
Obs.	244	244	244	244	244	244
Adj. R^2	0.705	0.941	0.554	0.76	0.899	0.566
Dev. Exp.	71.7%	94.4%	56.3%	76.8%	90.4%	59.3%
AIC	-1333.5	-1971.2	-1315.4	-1379.5	-1748.5	-1386.3

Table 19: Univariate GAM with interactions, CEF output gap change

	Canada	Japan	Norway	Sweden	Switzerland	USA
constant	0.0482791	0.0115984	0.0693832	0.0480811	0.0245492	0.0399078
	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***	(<2e-16)***
$s(E_t\pi_{t+1})$	2.86	8.90	2.89	5.96	7.58	3.63
	(< 2e-16)***	(<2e-16)***	(< 2e-16)***	(<2e-16)***	(< 2e-16)***	(4.86e-13)***
$s(E_t x_{t+1})$	1.00	2.84	2.47	1.00	1.00	5.98
	(0.112)	(1.92e-05)***	(0.2114)	(0.02112)**	(0.19659)	(0.001591)***
$s(E_t\pi_{t+1} \times t)$	3.30	7.89	8.77	8.44	3.91	8.79
	(< 2e-16)***	(< 2e-16)***	(1.61e-11)***	(< 2e-16)***	(0.00266)***	(< 2e-16)***
$s(E_t x_{t+1} \times t)$	5.74	4.50	2.04	2.24	5.18	5.82
	(1.22e-10)***	(2.71e-06)***	(0.0086)***	(0.00533)***	(3.21e-06)***	(5.94e-06)***
$s(E_t\pi_{t+1} \times E_t x_{t+1})$	8.03	7.03	1.00	1.00	1.00	5.27
	(2.47e-06)***	(0.169)	(0.8162)	(0.05084)*	(0.60364)	(0.000154)***
Obs.	244	244	244	244	244	244
Adj. R^2	0.873	0.984	0.694	0.923	0.913	0.781
Dev. Exp.	88.4%	98.6%	71.6%	92.9%	92%	80.8%
AIC	-1527.2	-2268.6	-1395.6	-1647.0	-1778.7	-1540.3

Note: The tables report the estimated degrees of freedom of the smooth functions from the GAM specifications. The latter are functions of economic fundamentals, $E_t\pi_{t+1}$ and $E_t x_{t+1}$. The last specification also includes interactions between the inflation and real output growth forecasts as well as interactions between the economic fundamentals and time. P-values are reported in parentheses. Superscripts (***) , (**) and (*) indicate significance level at 99%, 95% and 90% respectively.

Table 20: Root Forecast Mean Squared Error, CEF output gap change

Forecasting models	Canada	Japan	Norway	Sweden	Switzerland	USA
Bivariate GAM 1	0.05307126	***0.004222526***	0.01904761	0.02726679	0.004743018	***0.0194076***
Bivariate GAM 2	0.01766682	0.003877476	0.01644799	0.0163452	0.007366377	***0.01543891***
Univariate GAM 1	0.04315998	0.01541089	***0.01543967***	0.0252406	0.01379929	0.0303999
Univariate GAM 2	0.01761453	0.004600779	0.01647763	0.01651555	0.007772411	0.01578384
Augmented GAM 1	***0.005608583***	0.01249822	0.03081534	***0.01812005***	0.01914525	0.03306923
Augmented GAM 2	***0.01008813***	***0.002959573***	***0.01388107***	***0.006051951***	***0.005591748***	0.02010423
Linear model 1	0.03623399	0.03294018	0.02872529	0.03075535	***0.00265843***	0.02540582
Linear model 2	0.01859741	0.008989286	0.02208549	0.01715441	0.008430026	0.01861931
Forecast Gain 1	41.737	60.857	3.461	2.881	0.314	1.714
Forecast Gain 2	3.398	9.226	2.531	8.035	2.273	1.454

Note: The table reports the out-of-sample forecasting performance of the semi-parametric and linear models. For each model the first row displays the RFMSE of the forecasts from December 2009 to March 2010, while the second row indicates the corresponding RFMSE for the rolling window forecasts. The Univariate GAM refers to the model without interactions, while the Augmented GAM corresponds to the model with the relevant interaction terms. The best forecasting performance is reported in bold characters. The forecast gain is defined as the ratio between the forecast mean squared error of the linear model and the one of the best semi-parametric model for each country as indicated in bold characters.

9.5 Figures

9.5.1 Bivariate GAM, OECD output growth forecasts

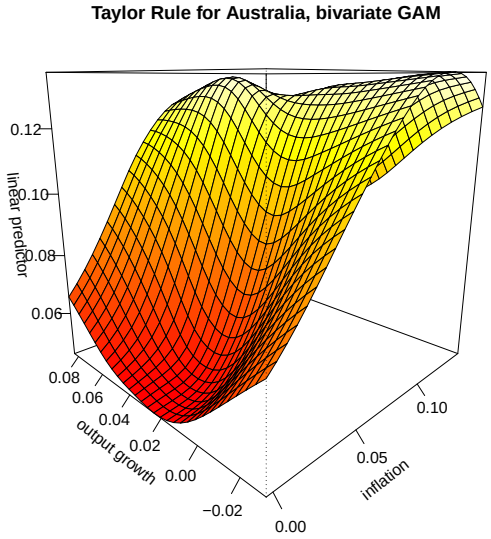


Figure 9: Estimated Surface of the Policy Rate for Australia, output growth

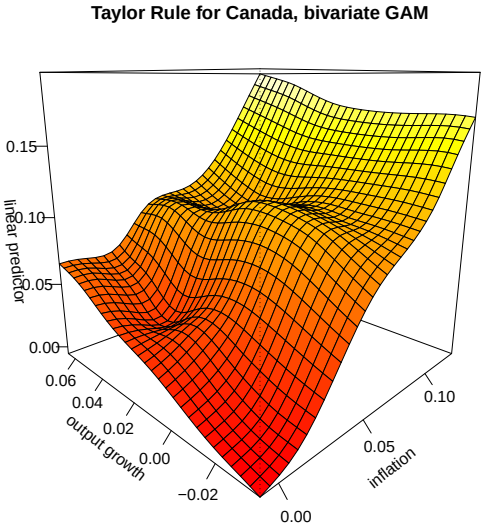


Figure 10: Estimated Surface of the Policy Rate for Canada, output growth

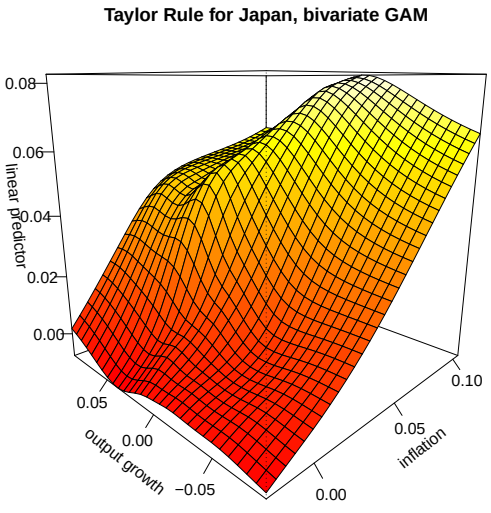


Figure 11: Estimated Surface of the Policy Rate for Japan, output growth

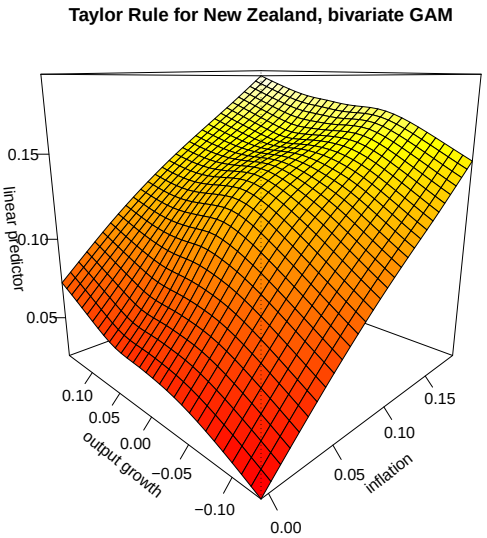


Figure 12: Estimated Surface of the Policy Rate for New Zealand, output growth

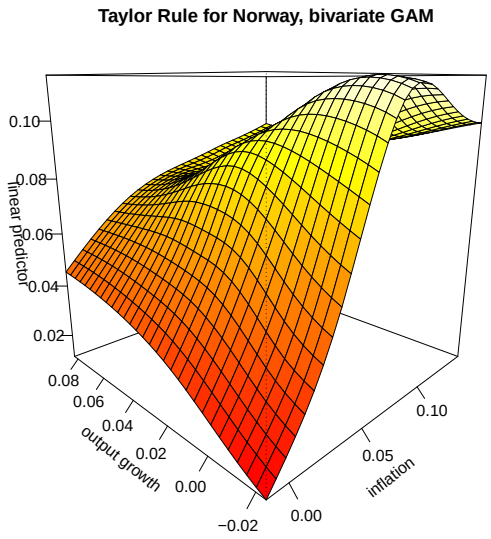


Figure 13: Estimated Surface of the Policy Rate for Norway, output growth

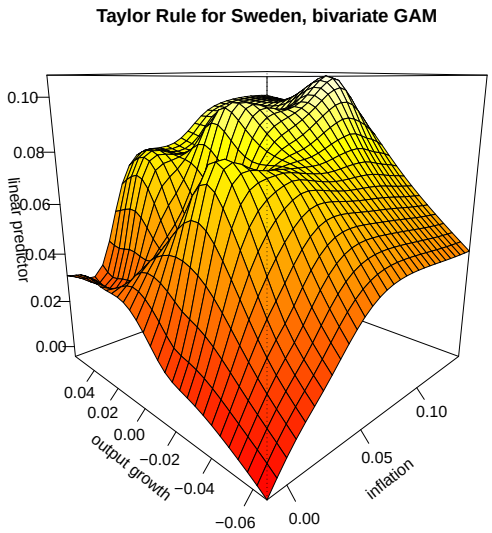


Figure 14: Estimated Surface of the Policy Rate for Sweden, output growth

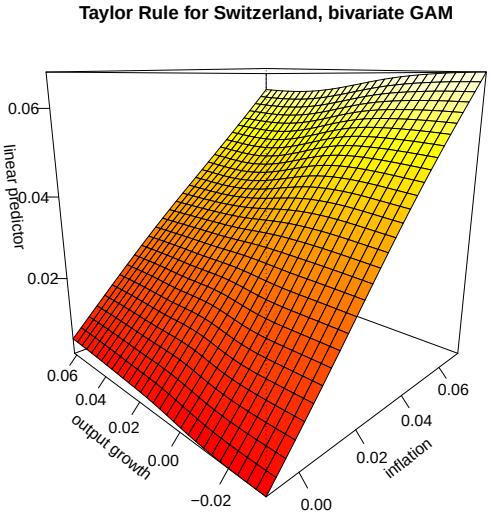


Figure 15: Estimated Surface of the Policy Rate for Switzerland, output growth

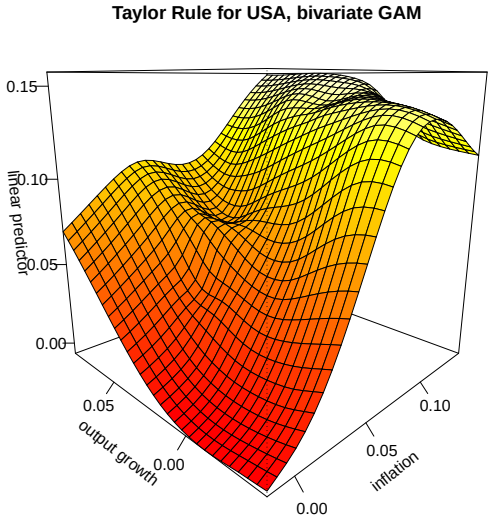


Figure 16: Estimated Surface of the Policy Rate for USA, output growth

9.5.2 Bivariate GAM, OECD output gap change forecasts

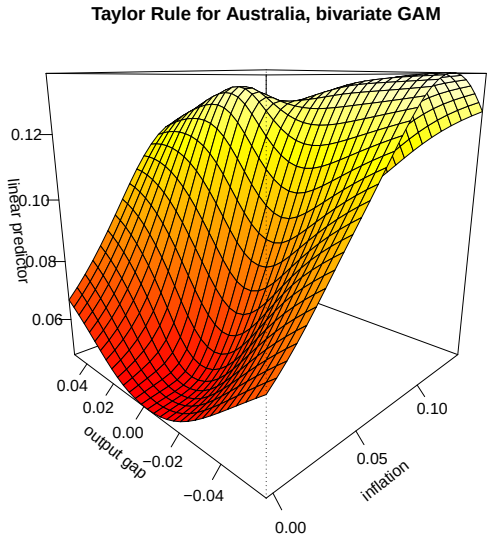


Figure 17: Estimated Surface of the Policy Rate for Australia, output gap

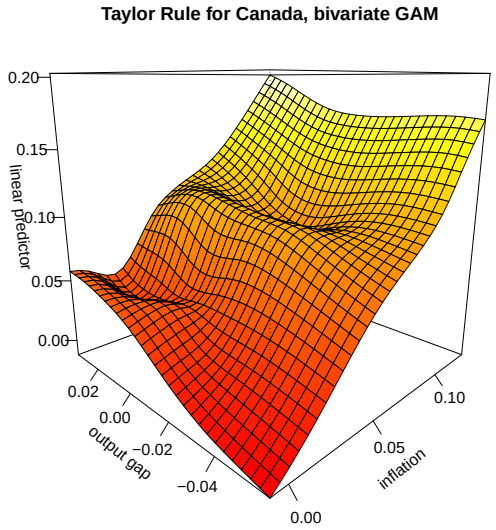


Figure 18: Estimated Surface of the Policy Rate for Canada, output gap

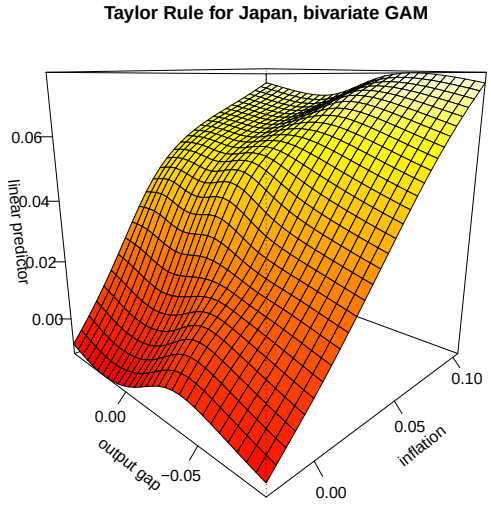


Figure 19: Estimated Surface of the Policy Rate for Japan, output gap

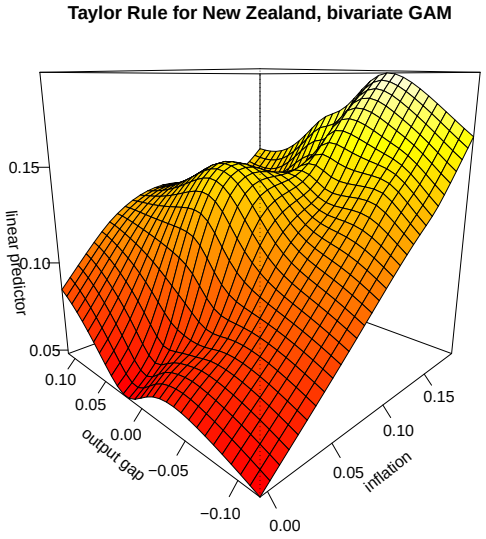


Figure 20: Estimated Surface of the Policy Rate for New Zealand, output gap

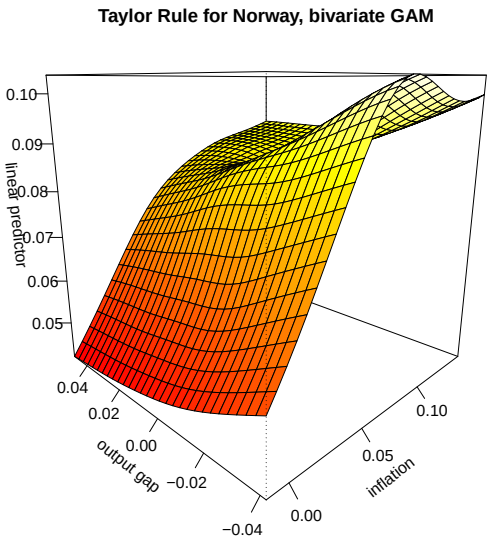


Figure 21: Estimated Surface of the Policy Rate for Norway, output gap

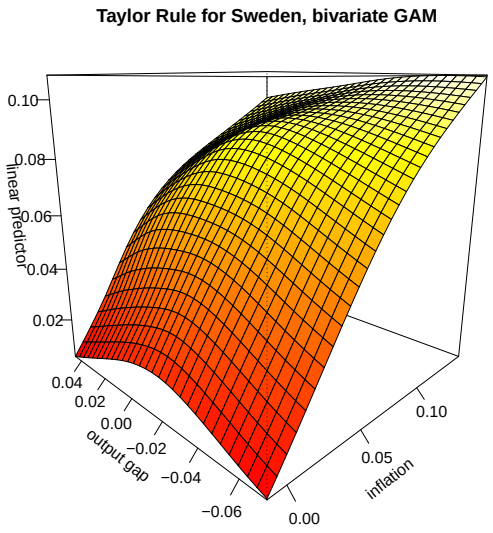


Figure 22: Estimated Surface of the Policy Rate for Sweden, output gap

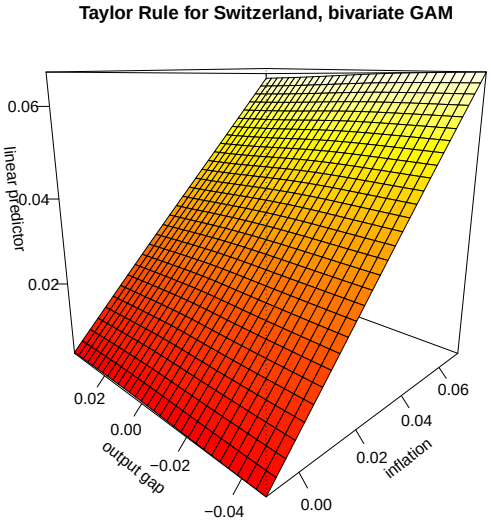


Figure 23: Estimated Surface of the Policy Rate for Switzerland, output gap

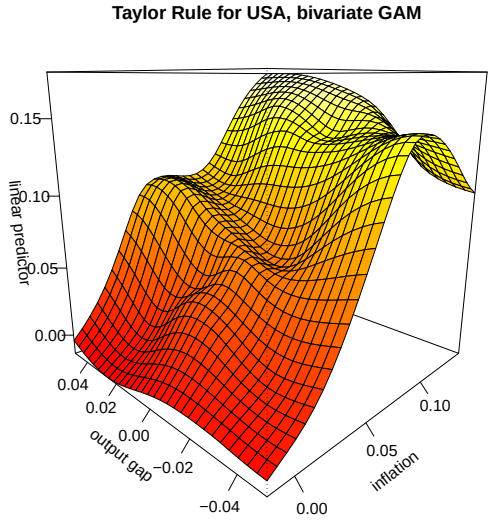


Figure 24: Estimated Surface of the Policy Rate for USA, output gap

9.5.3 Bivariate GAM, Consensus output growth forecasts

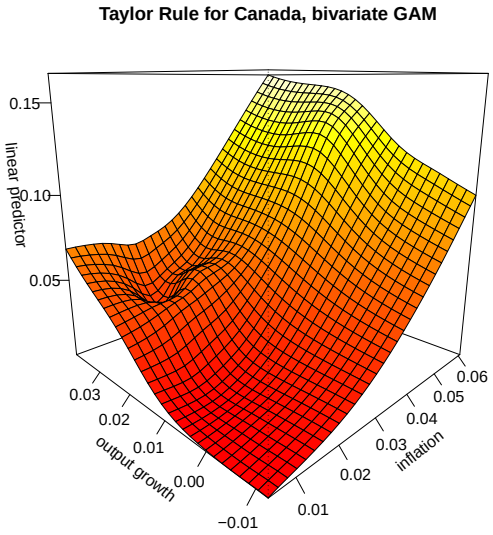


Figure 25: Estimated Surface of the Policy Rate for Canada, CEF output growth

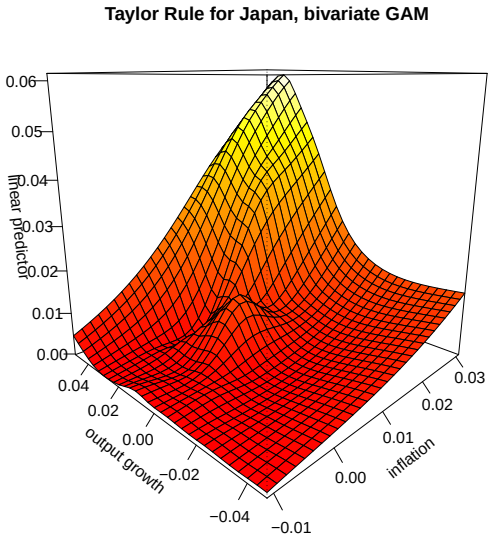


Figure 26: Estimated Surface of the Policy Rate for Japan, CEF output growth

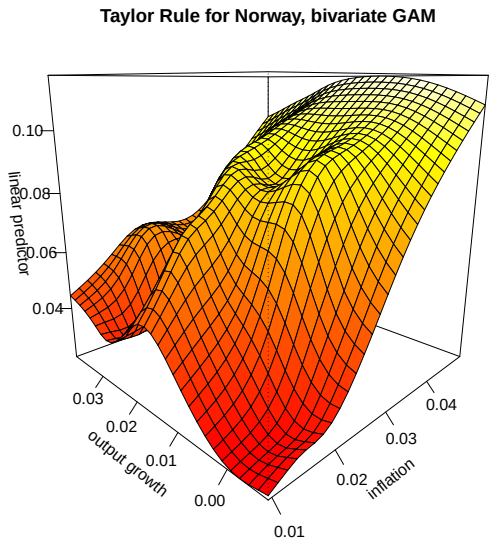


Figure 27: Estimated Surface of the Policy Rate for Norway, CEF output growth

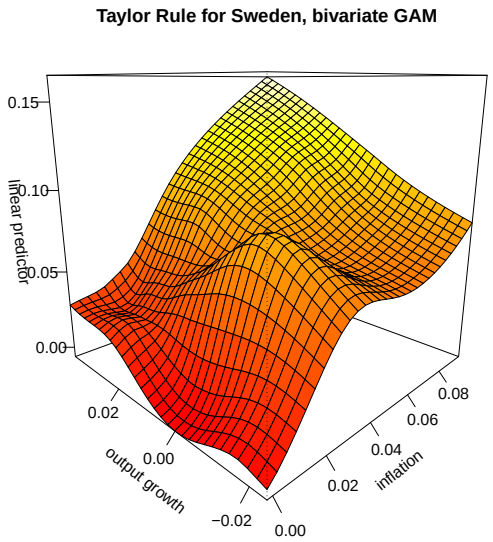


Figure 28: Estimated Surface of the Policy Rate for Sweden, CEF output growth

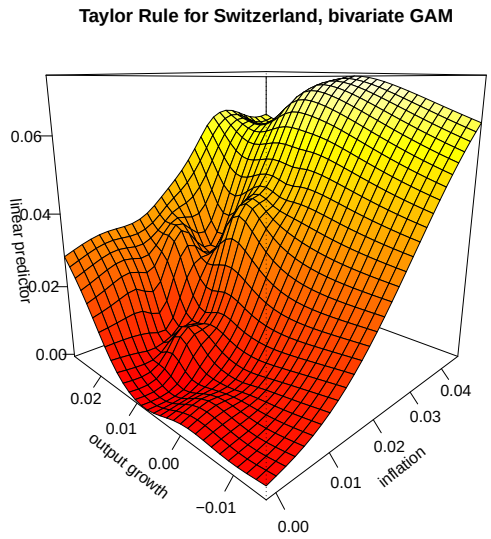


Figure 29: Estimated Surface of the Policy Rate for Switzerland, CEF output growth

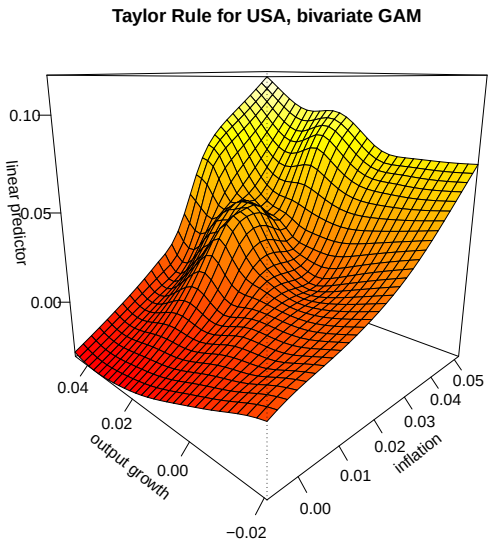


Figure 30: Estimated Surface of the Policy Rate for USA, CEF output growth

9.5.4 Bivariate GAM, Consensus output gap change forecasts

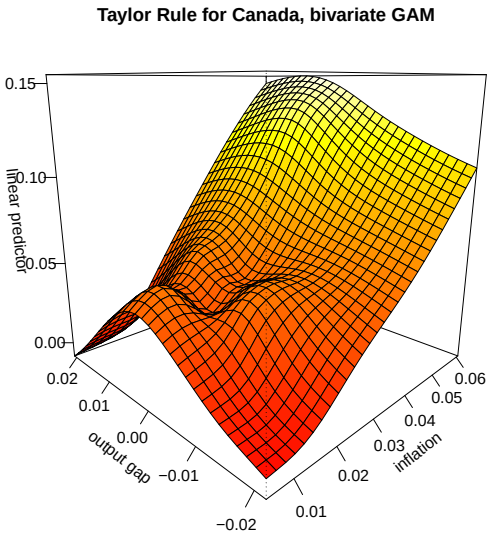


Figure 31: Estimated Surface of the Policy Rate for Canada, CEF output gap

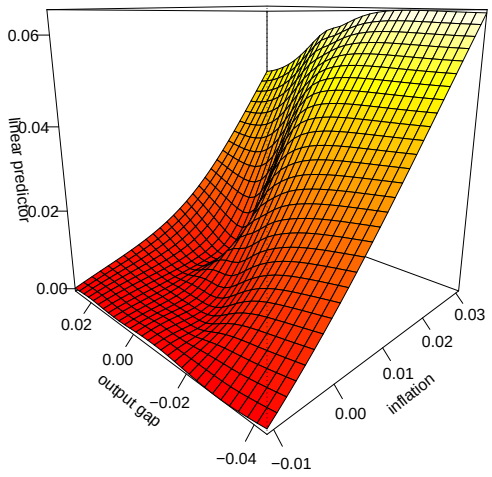


Figure 32: Estimated Surface of the Policy Rate for Japan, CEF output gap

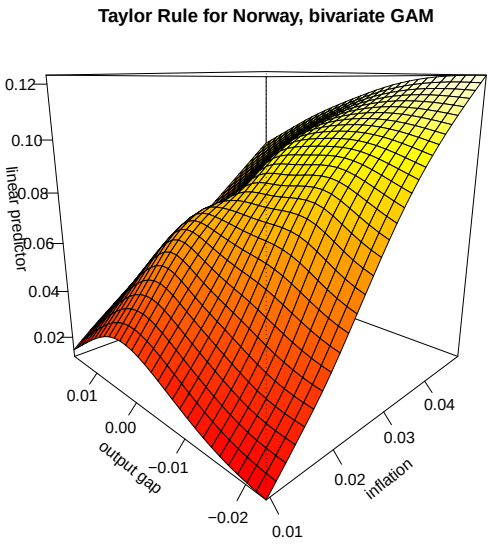


Figure 33: Estimated Surface of the Policy Rate for Norway, CEF output gap

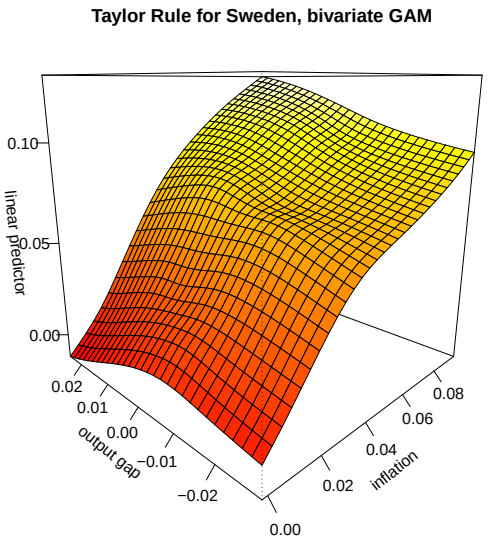


Figure 34: Estimated Surface of the Policy Rate for Sweden, CEF output gap

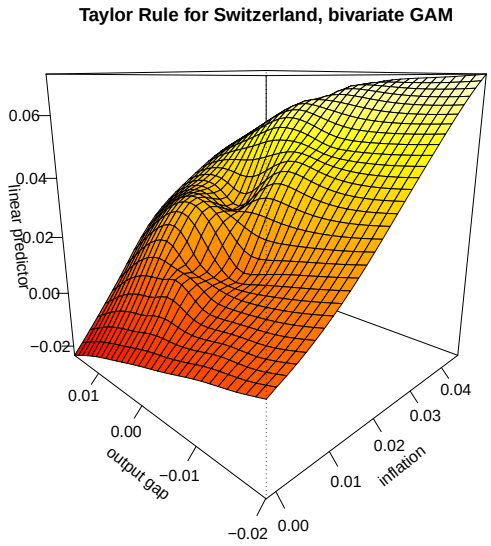


Figure 35: Estimated Surface of the Policy Rate for Switzerland, CEF output gap

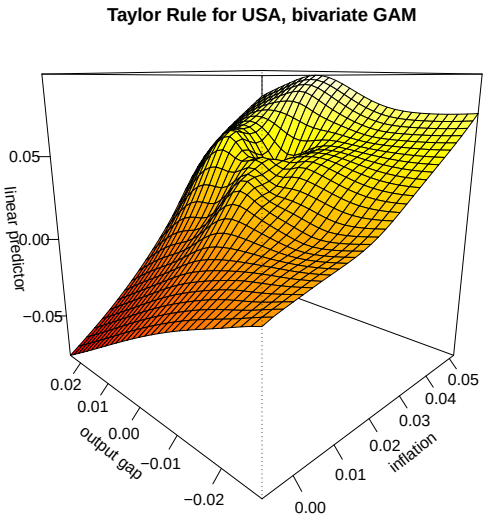


Figure 36: Estimated Surface of the Policy Rate for USA, CEF output gap

9.5.5 Baseline specifications, OECD forecasts

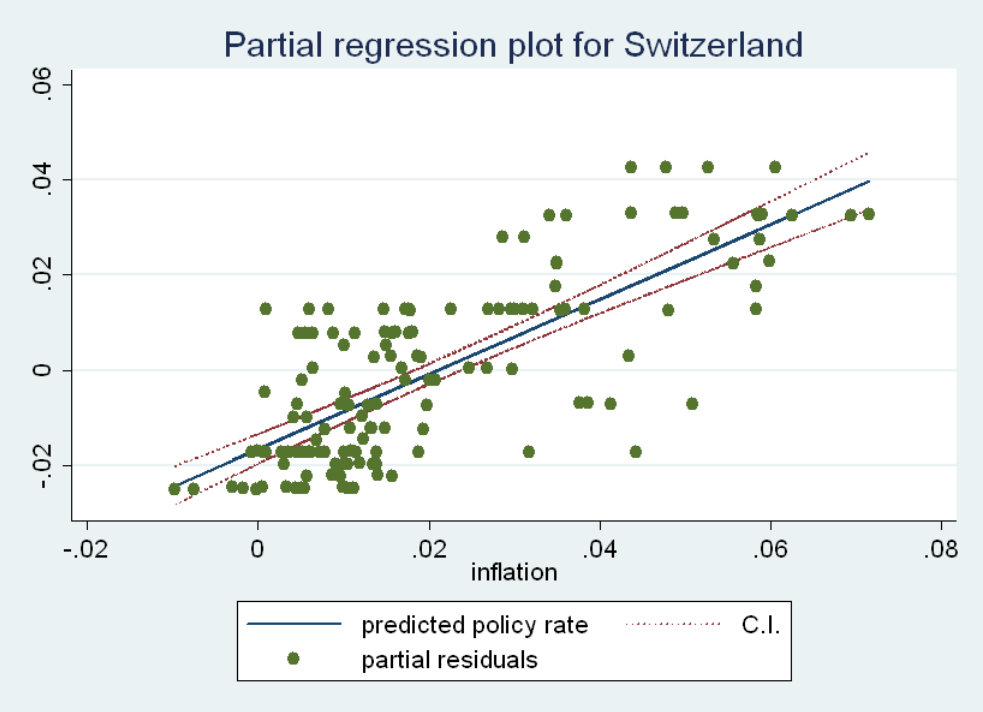


Figure 37: Regression plot with inflation, OLS method

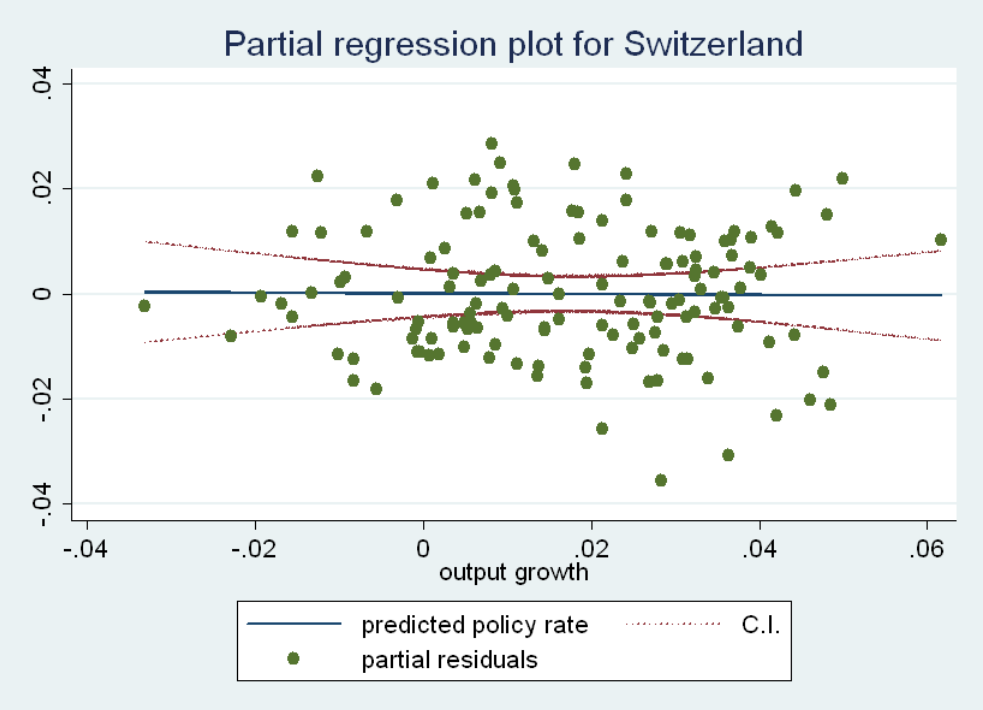
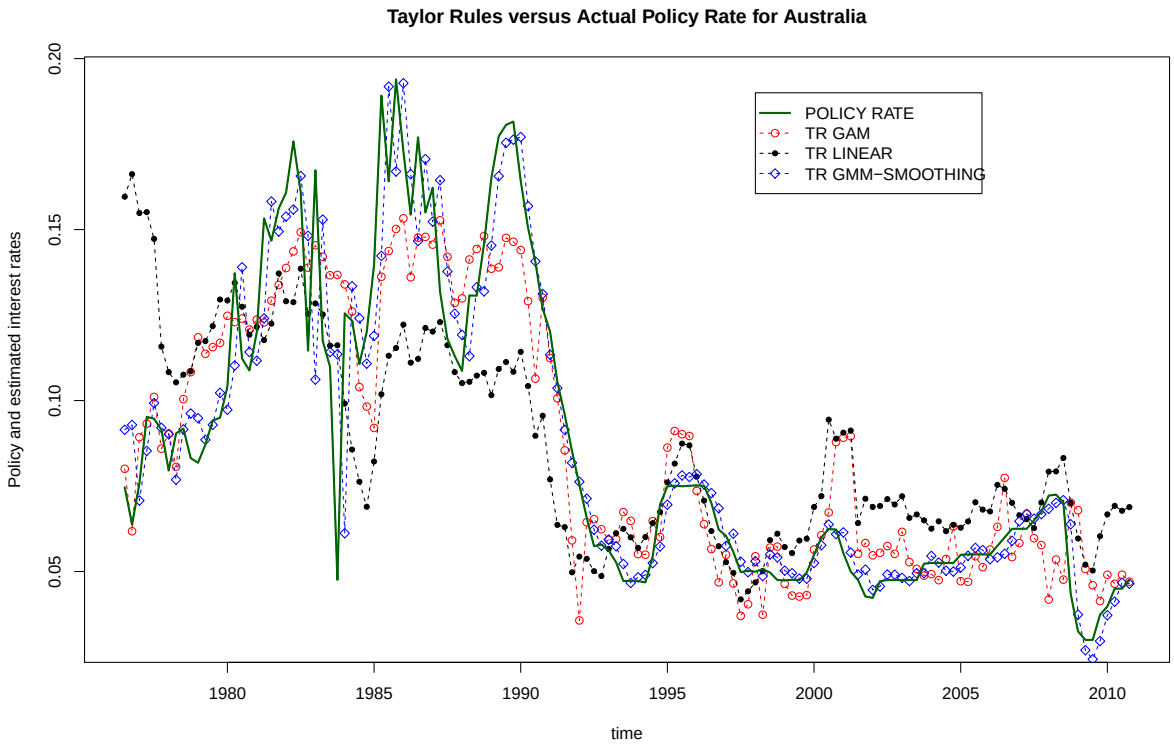
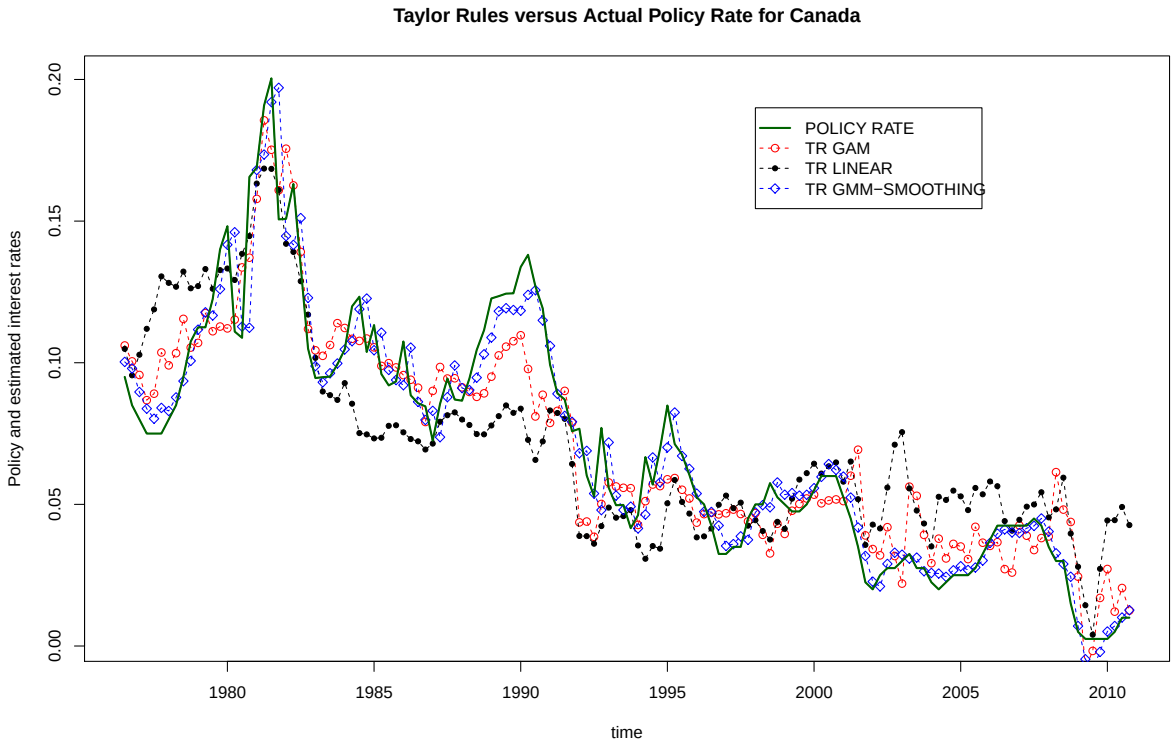


Figure 38: Regression plot with output growth, OLS method

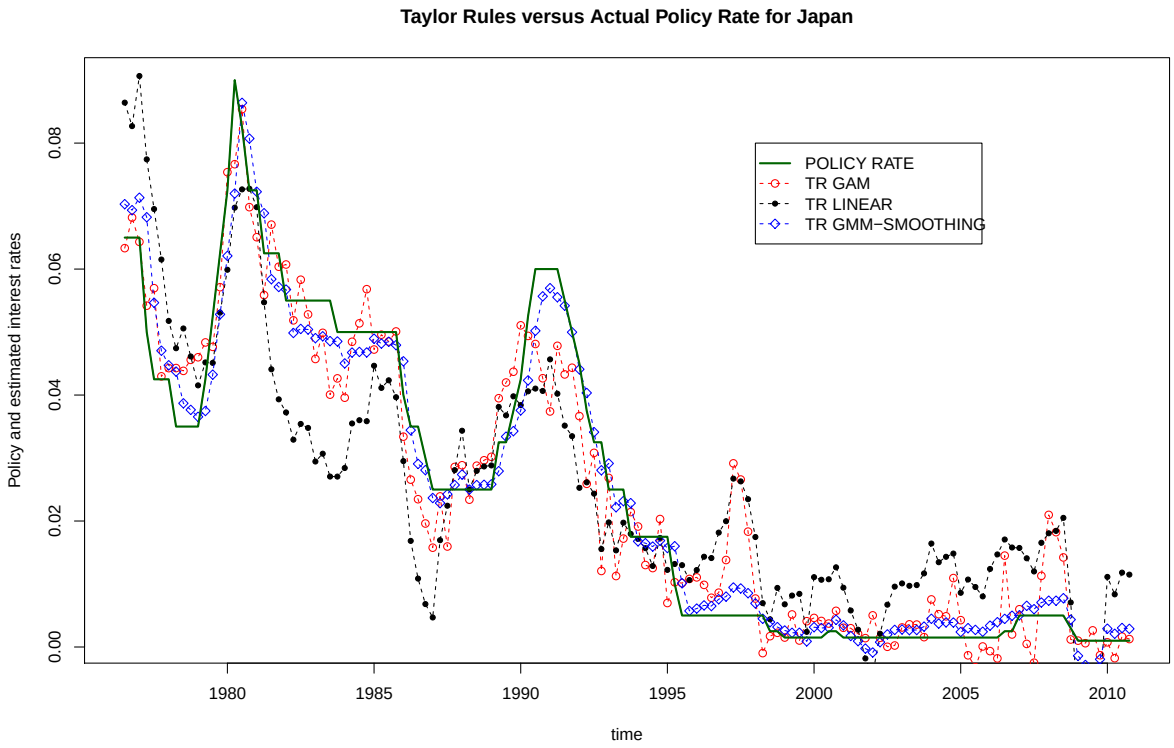
Figure 39: Fitted values against Actual Policy Rate



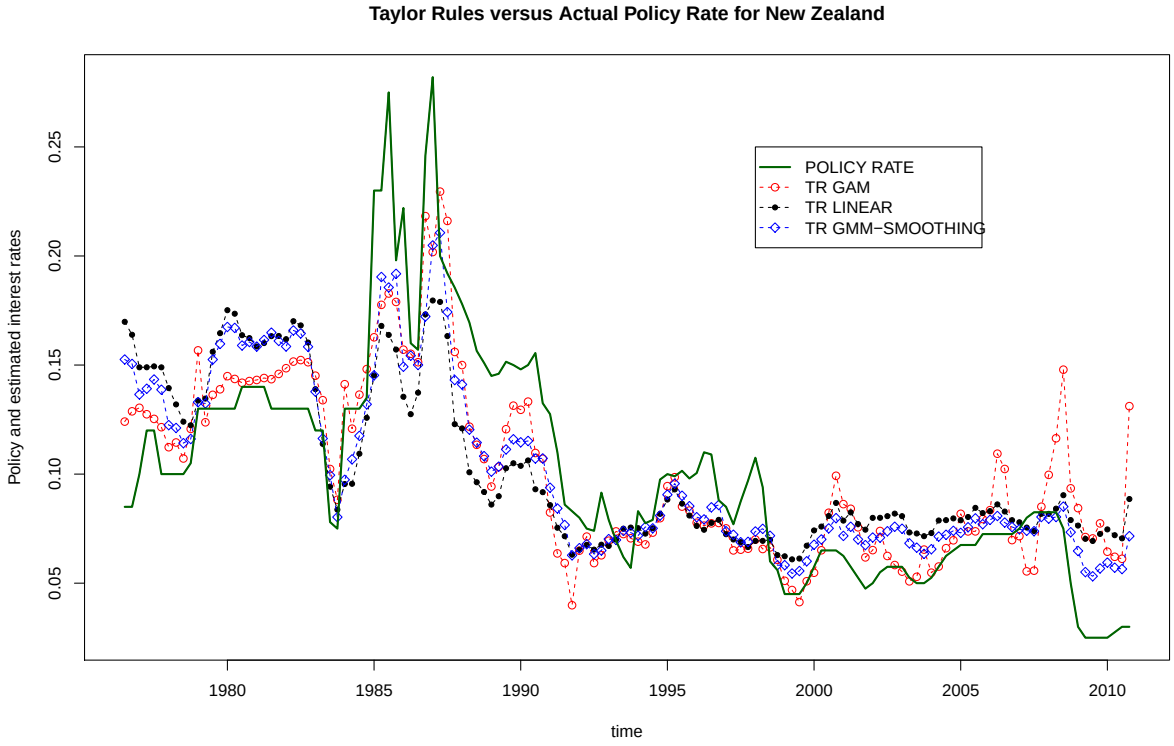
(a) Fitted against Actual Policy Rate for Australia



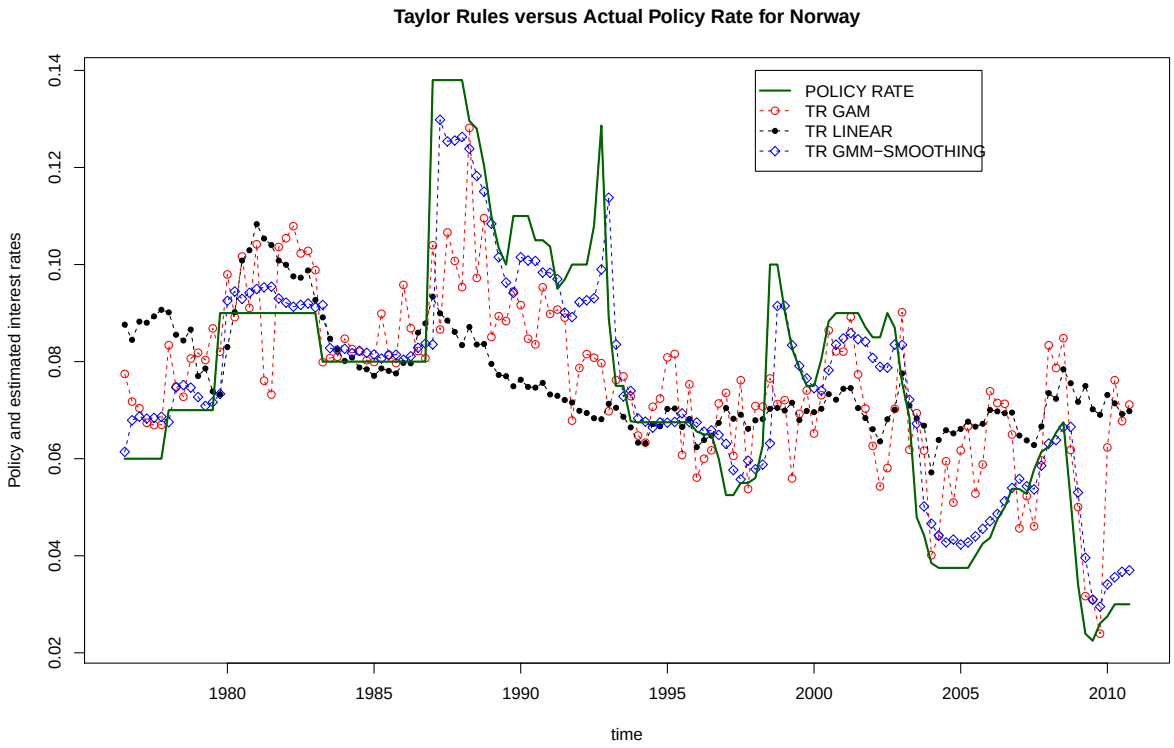
(b) Fitted against Actual Policy Rate for Canada



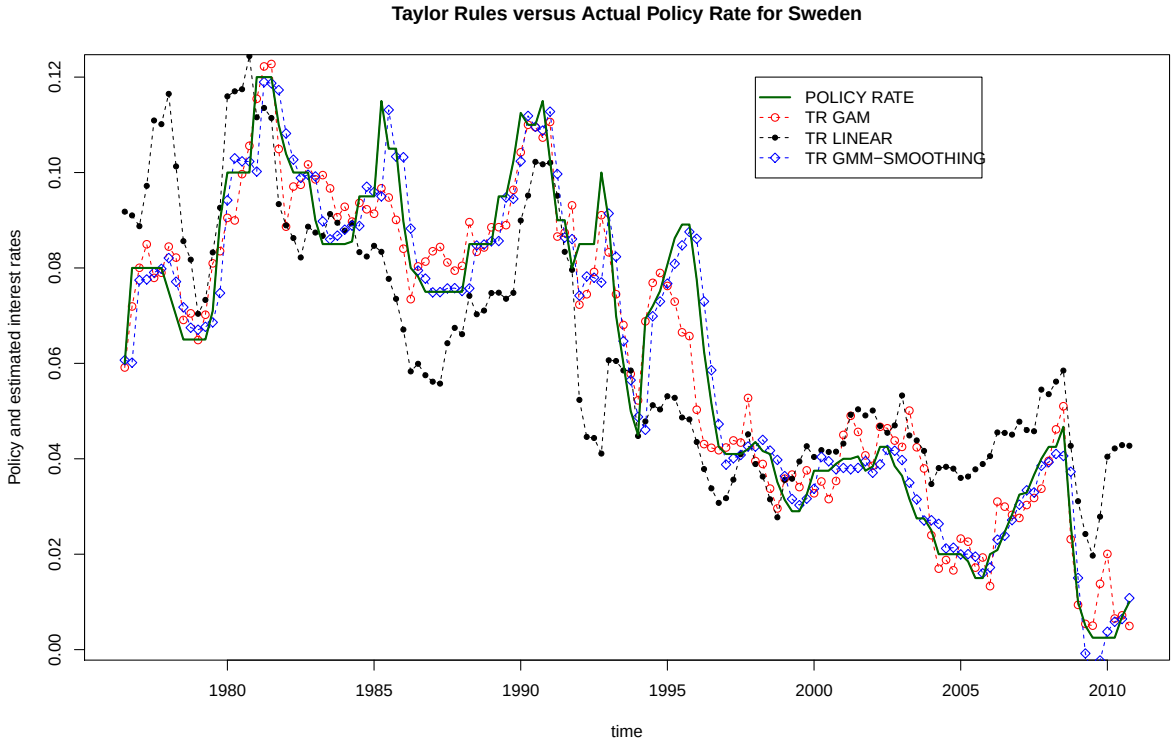
(c) Fitted against Actual Policy Rate for Japan



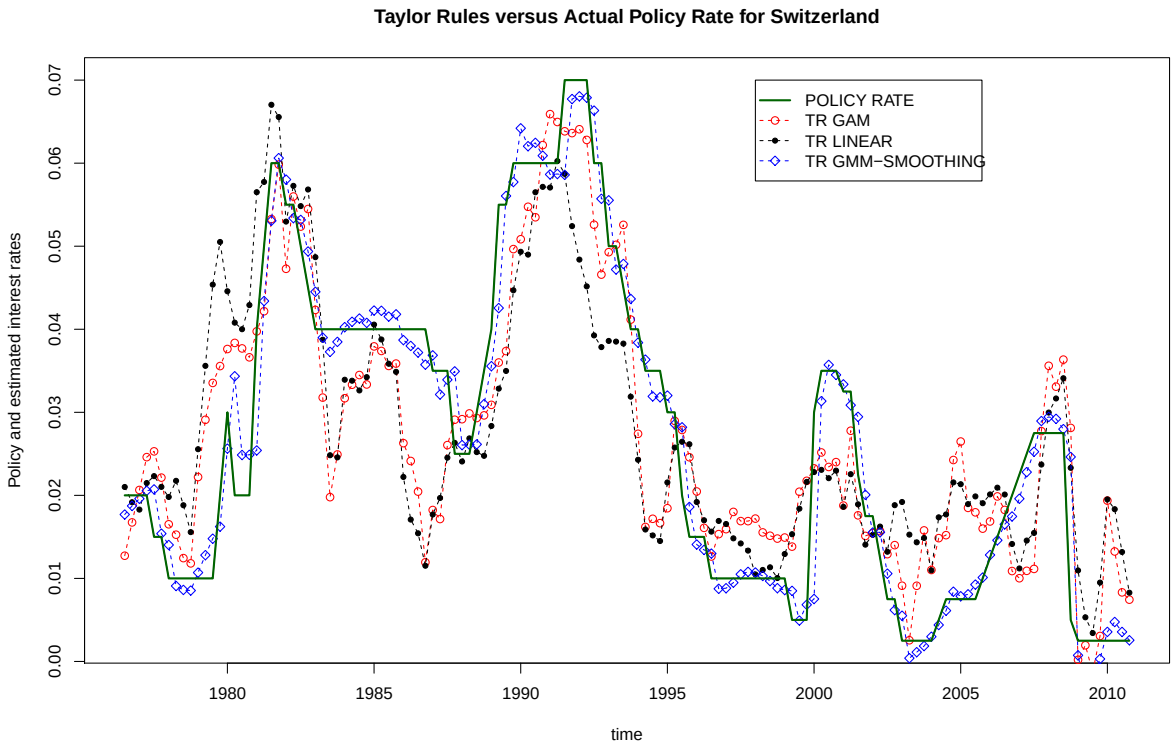
(d) Fitted against Actual Policy Rate for New Zealand



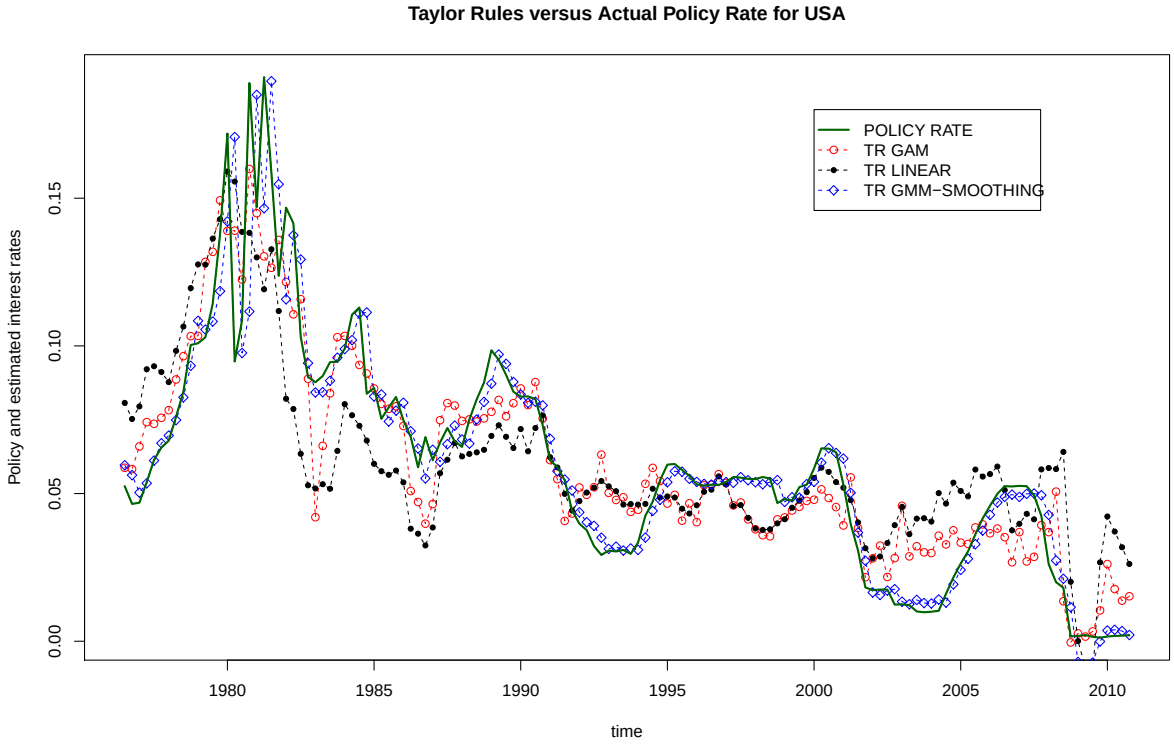
(e) Fitted against Actual Policy Rate for Norway



(f) Fitted against Actual Policy Rate for Sweden

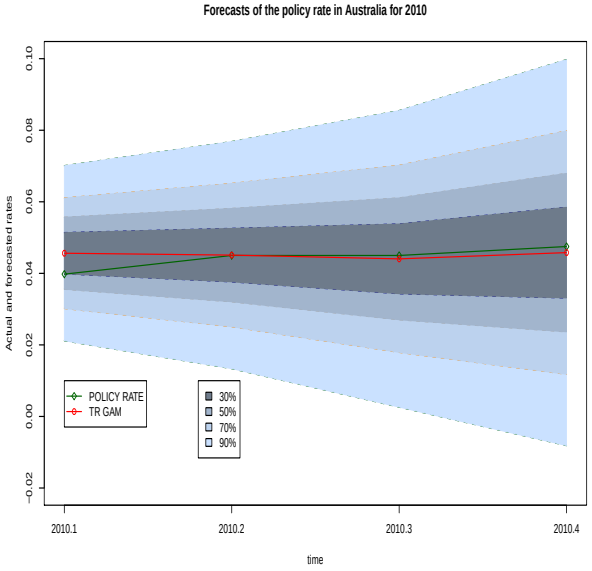


(g) Fitted against Actual Policy Rate for Switzerland

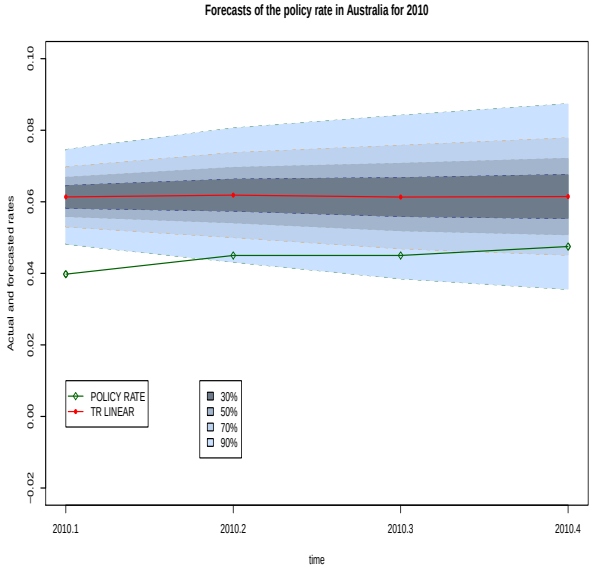


(h) Fitted against Actual Policy Rate for USA

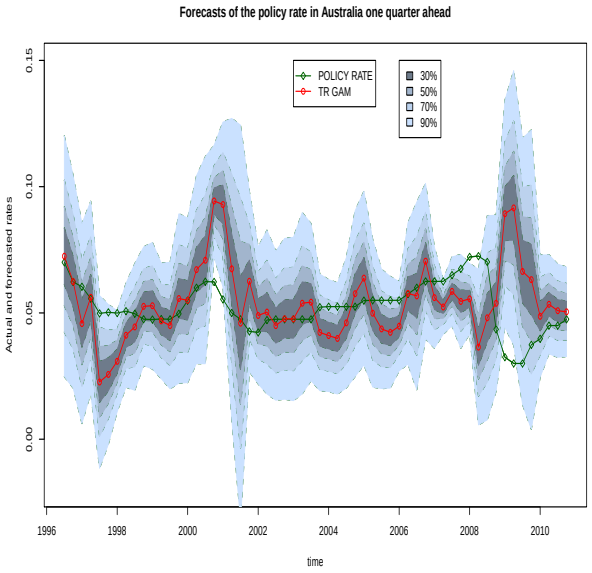
Figure 40: Out-of-sample forecasts of the policy rate in Australia



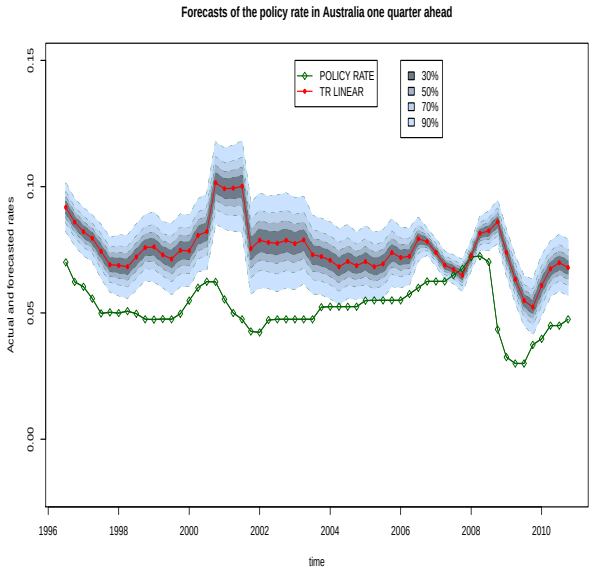
(a) Forecasts of the policy rate using the GAM Taylor Rule



(b) Forecasts of the policy rate using the Linear Taylor Rule

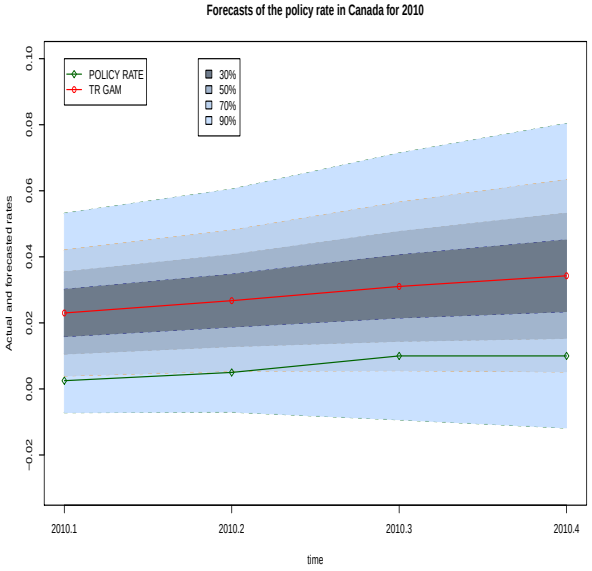


(c) Forecasts of the policy rate using the GAM Taylor Rule, rolling window

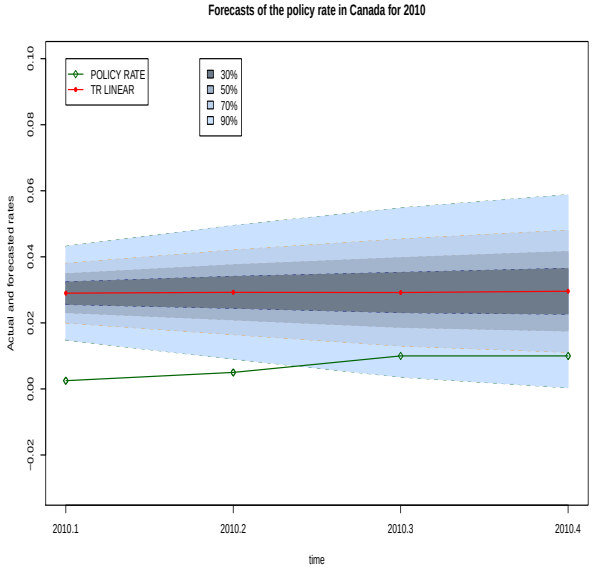


(d) Forecasts of the policy rate using the Linear Taylor Rule, rolling window

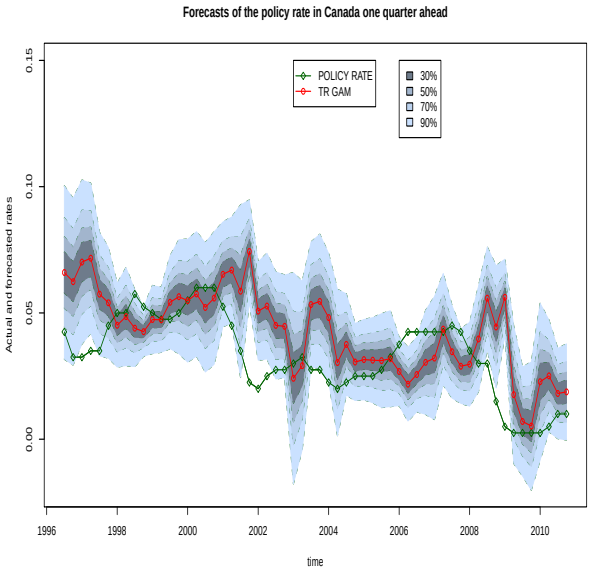
Figure 41: Out-of-sample forecasts of the policy rate in Canada



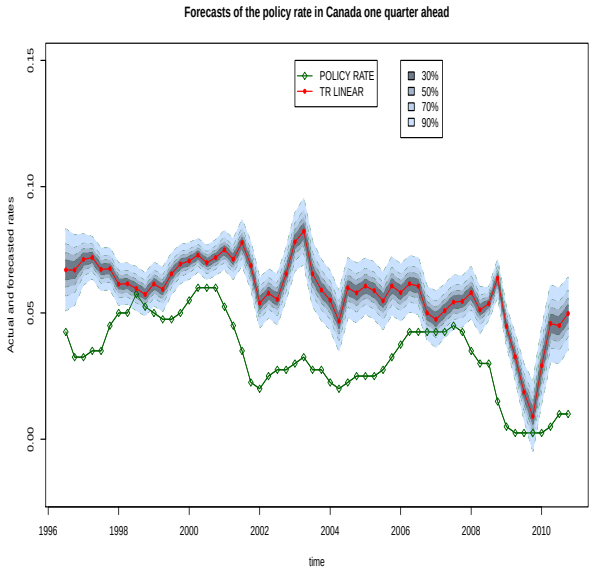
(a) Forecasts of the policy rate using the GAM Taylor Rule



(b) Forecasts of the policy rate using the Linear Taylor Rule

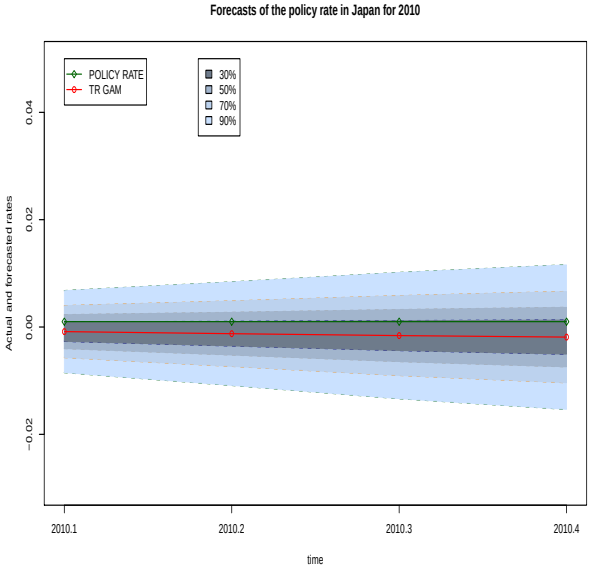


(c) Forecasts of the policy rate using the GAM Taylor Rule, rolling window

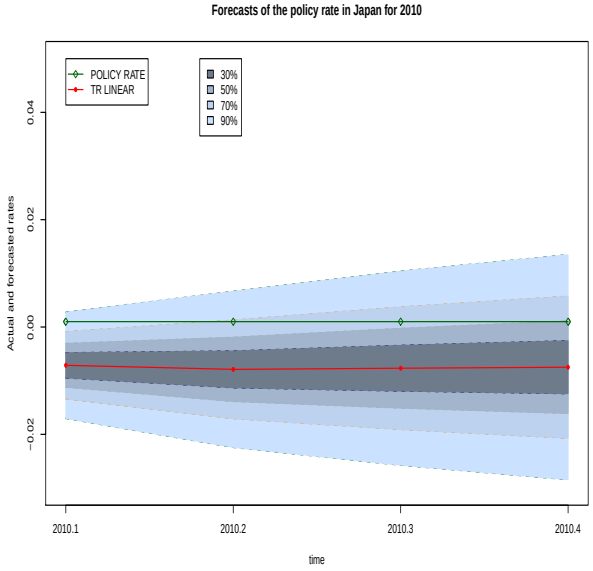


(d) Forecasts of the policy rate using the Linear Taylor Rule, rolling window

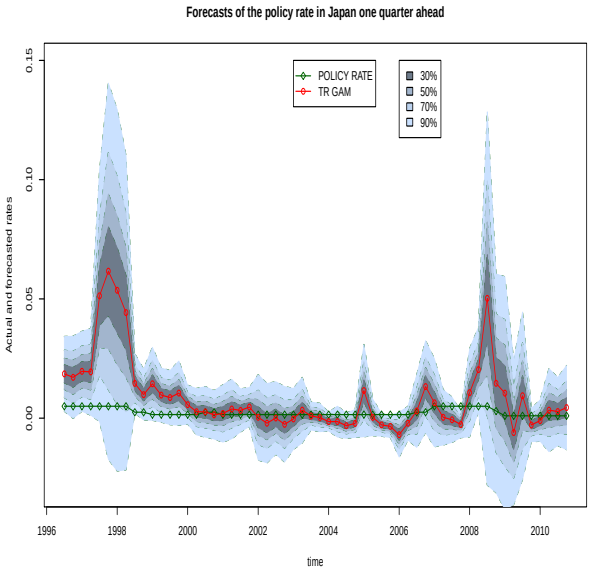
Figure 42: Out-of-sample forecasts of the policy rate in Japan



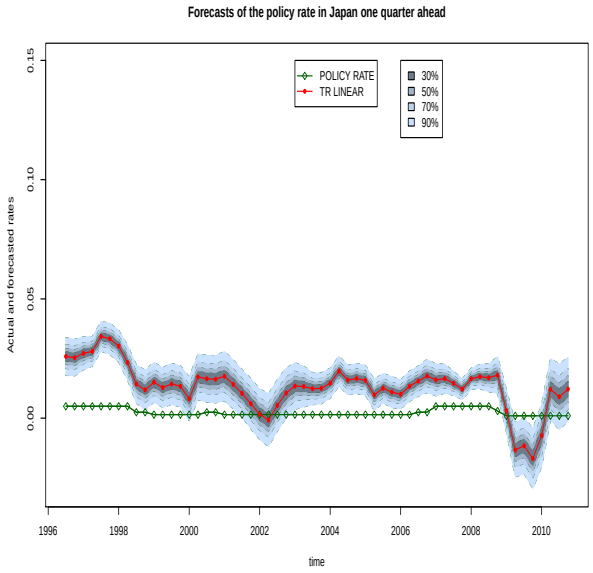
(a) Forecasts of the policy rate using the GAM Taylor Rule



(b) Forecasts of the policy rate using the Linear Taylor Rule

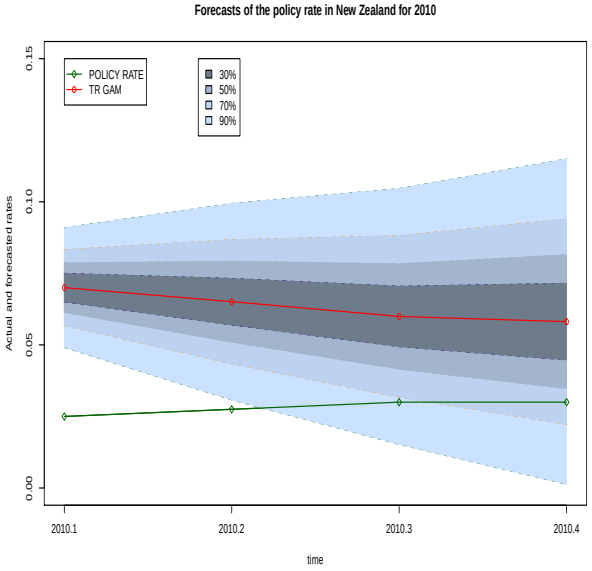


(c) Forecasts of the policy rate using the GAM Taylor Rule, rolling window

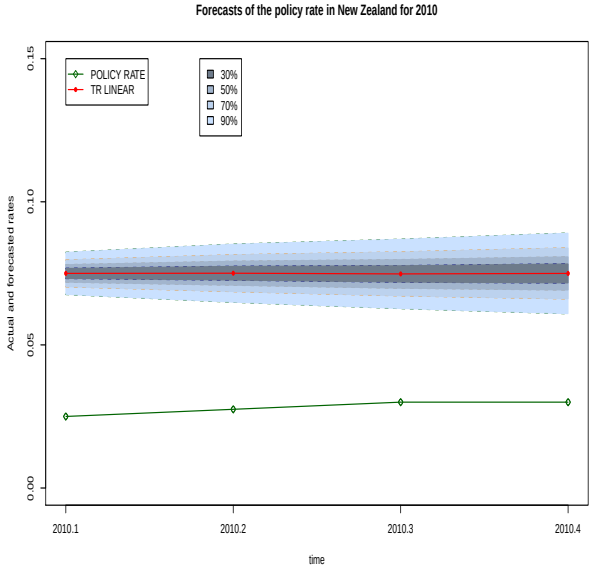


(d) Forecasts of the policy rate using the Linear Taylor Rule, rolling window

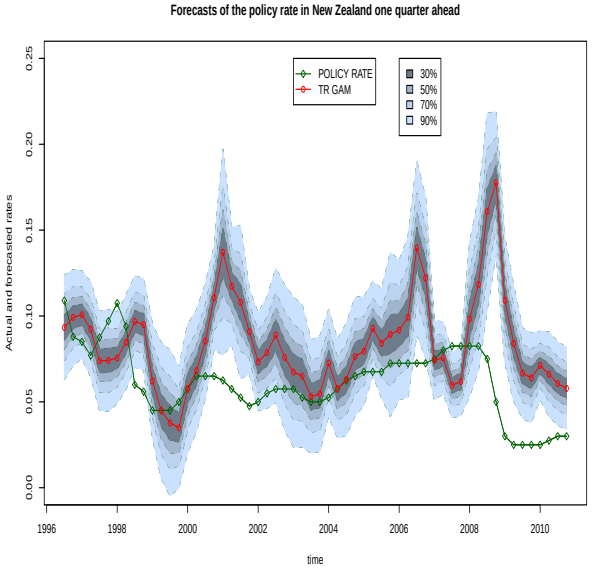
Figure 43: Out-of-sample forecasts of the policy rate in New Zealand



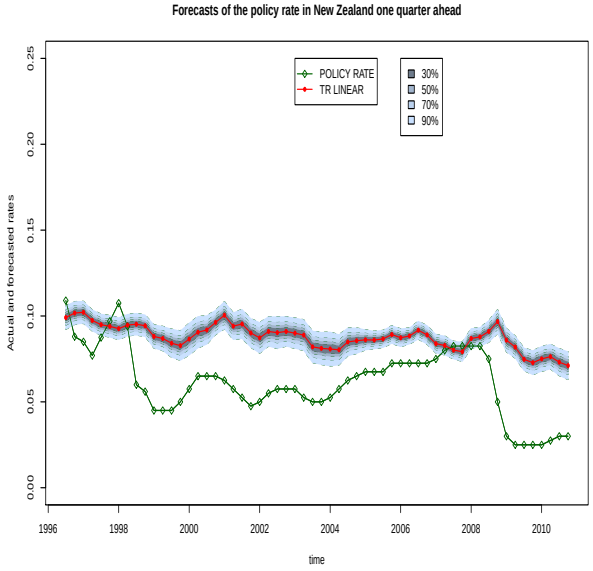
(a) Forecasts of the policy rate using the GAM Taylor Rule



(b) Forecasts of the policy rate using the Linear Taylor Rule

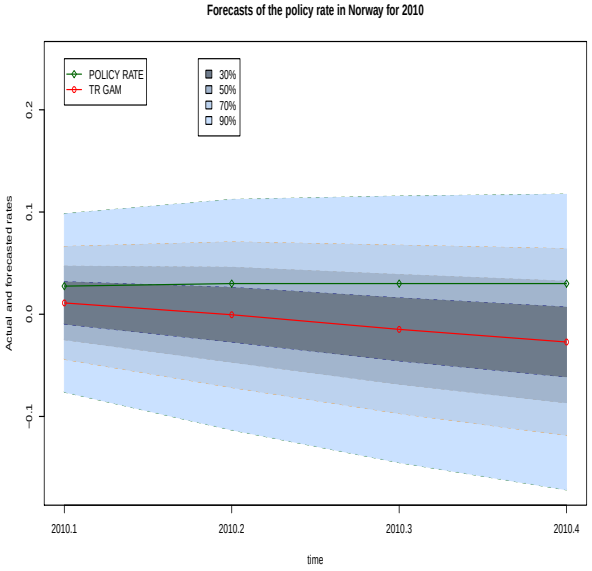


(c) Forecasts of the policy rate using the GAM Taylor Rule, rolling window

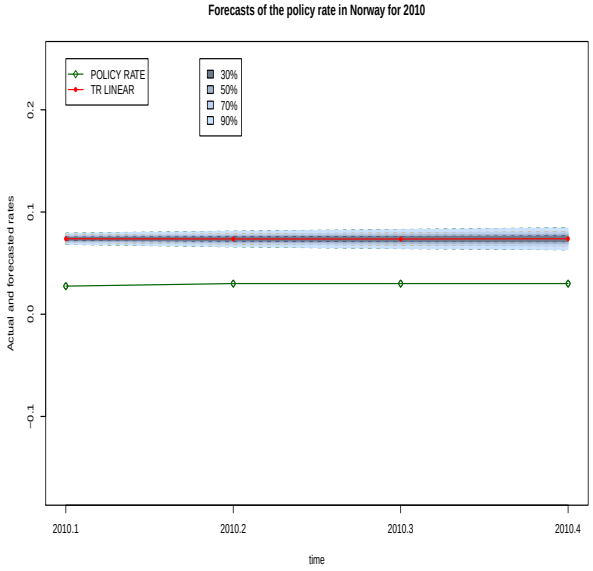


(d) Forecasts of the policy rate using the Linear Taylor Rule, rolling window

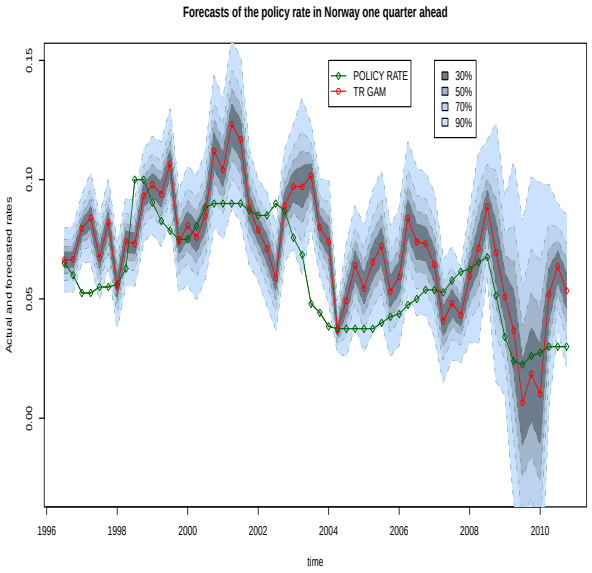
Figure 44: Out-of-sample forecasts of the policy rate in Norway



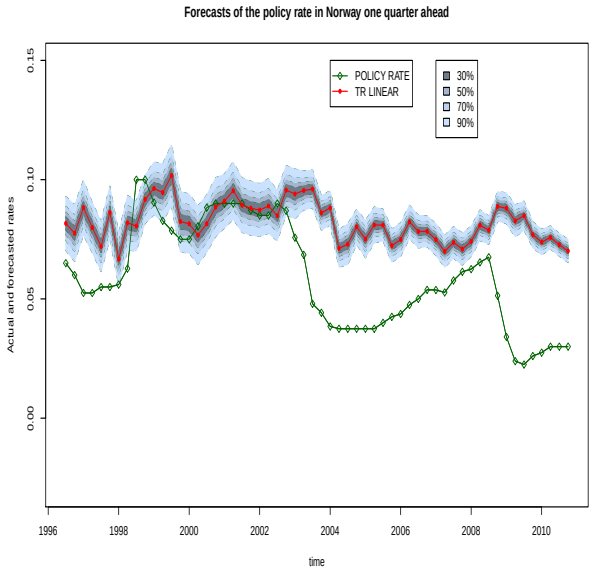
(a) Forecasts of the policy rate using the GAM Taylor Rule



(b) Forecasts of the policy rate using the Linear Taylor Rule

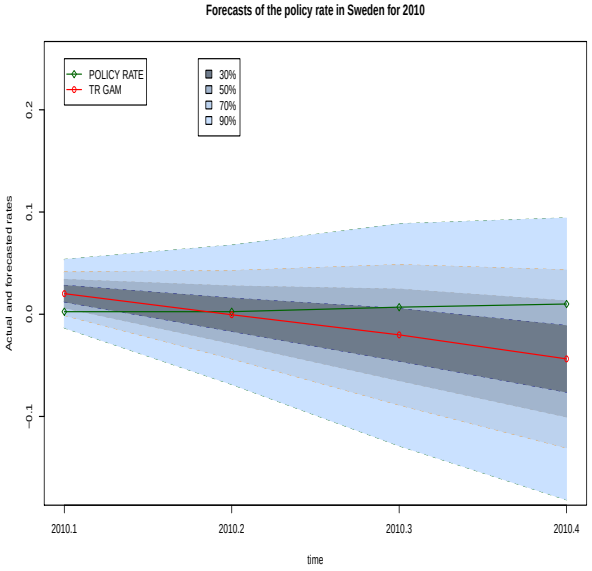


(c) Forecasts of the policy rate using the GAM Taylor Rule, rolling window

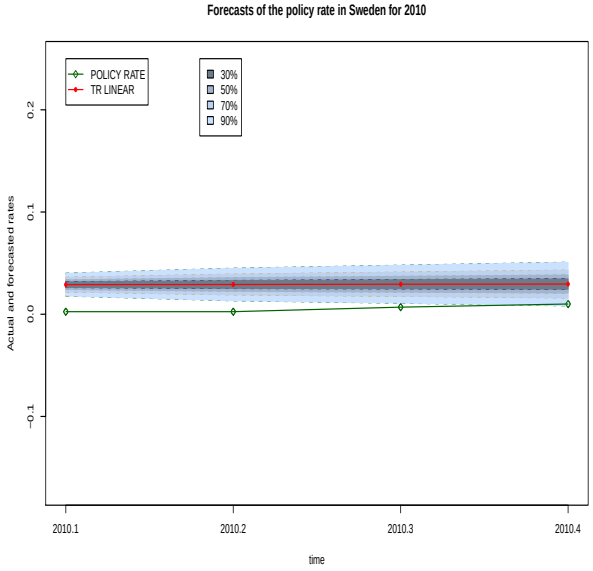


(d) Forecasts of the policy rate using the Linear Taylor Rule, rolling window

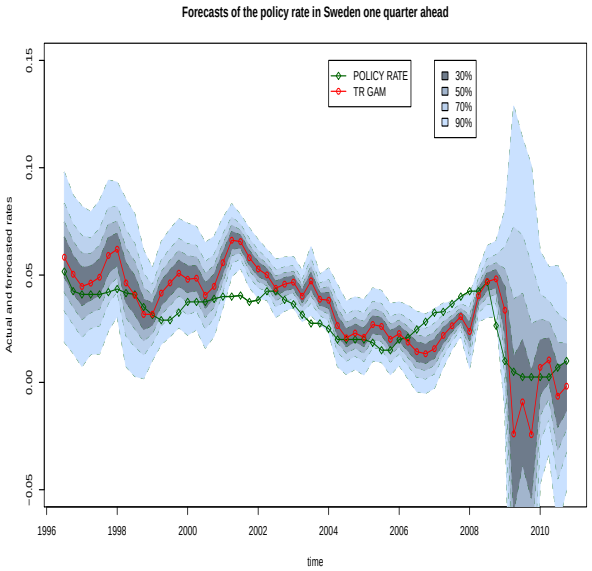
Figure 45: Out-of-sample forecasts of the policy rate in Sweden



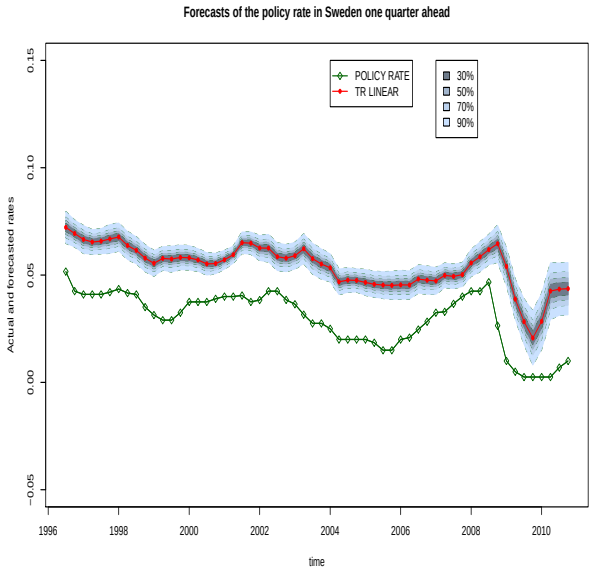
(a) Forecasts of the policy rate using the GAM Taylor Rule



(b) Forecasts of the policy rate using the Linear Taylor Rule

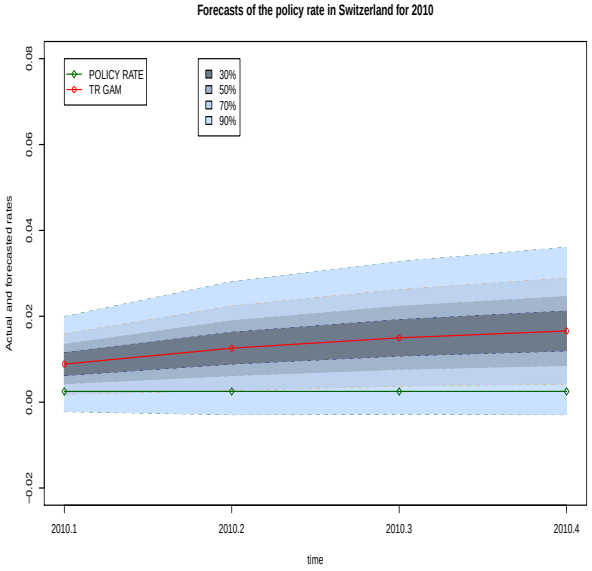


(c) Forecasts of the policy rate using the GAM Taylor Rule, rolling window

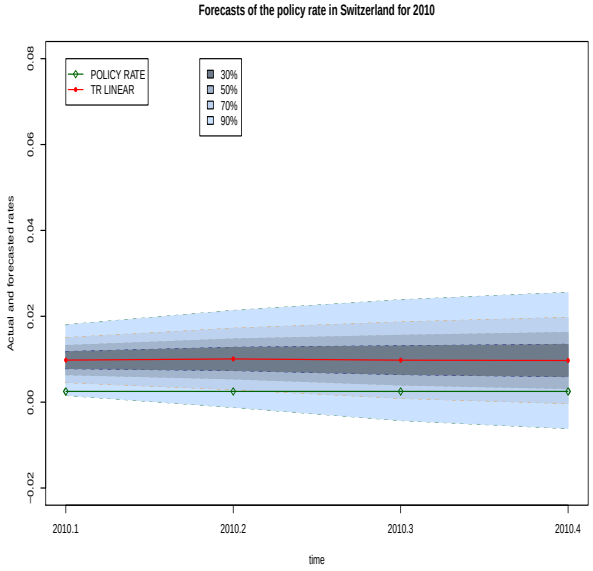


(d) Forecasts of the policy rate using the Linear Taylor Rule, rolling window

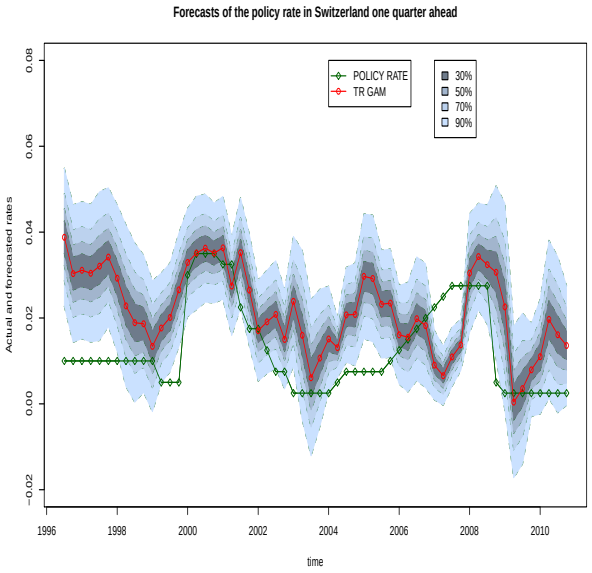
Figure 46: Out-of-sample forecasts of the policy rate in Switzerland



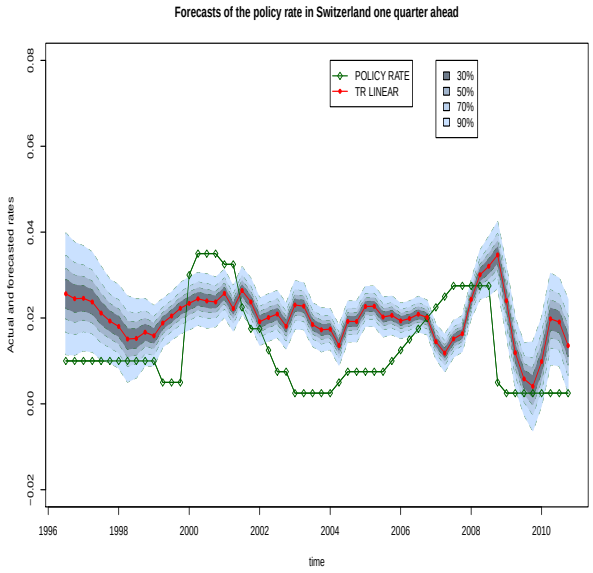
(a) Forecasts of the policy rate using the GAM Taylor Rule



(b) Forecasts of the policy rate using the Linear Taylor Rule

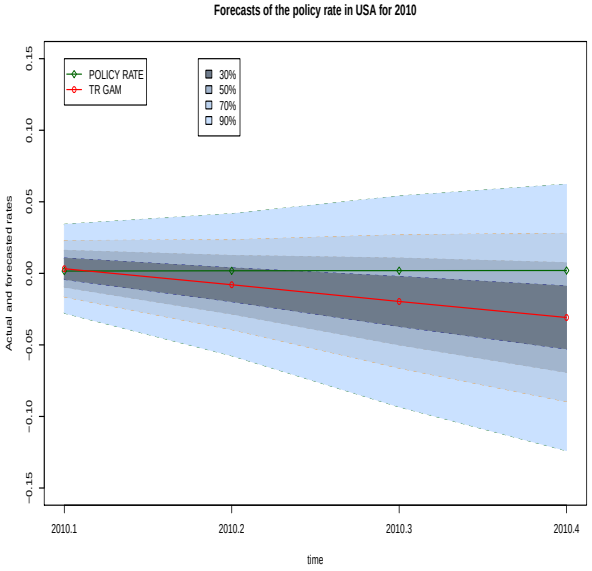


(c) Forecasts of the policy rate using the GAM Taylor Rule, rolling window

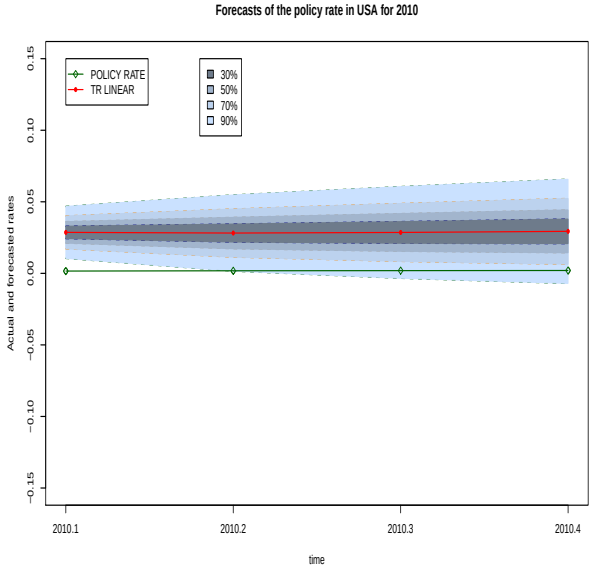


(d) Forecasts of the policy rate using the Linear Taylor Rule, rolling window

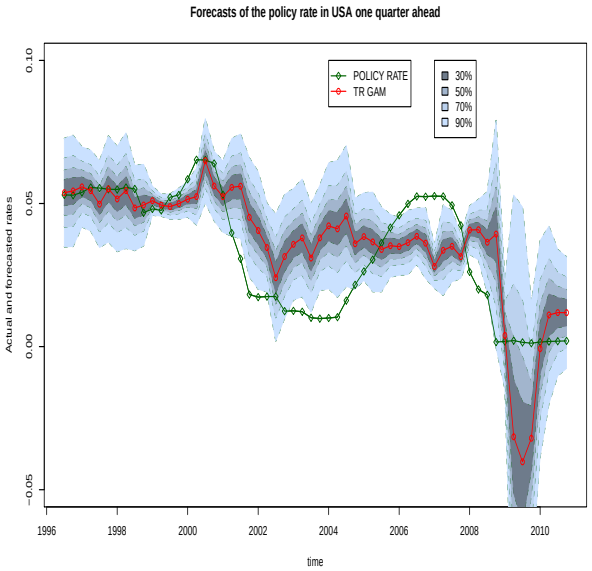
Figure 47: Out-of-sample forecasts of the policy rate in USA



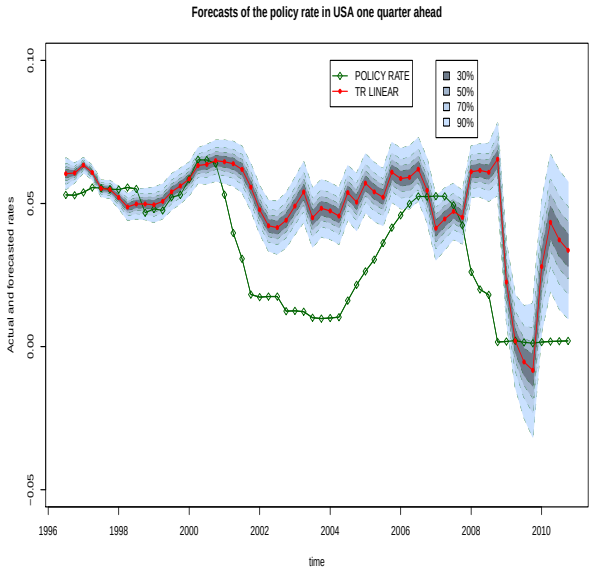
(a) Forecasts of the policy rate using the GAM Taylor Rule



(b) Forecasts of the policy rate using the Linear Taylor Rule



(c) Forecasts of the policy rate using the GAM Taylor Rule, rolling window



(d) Forecasts of the policy rate using the Linear Taylor Rule, rolling window